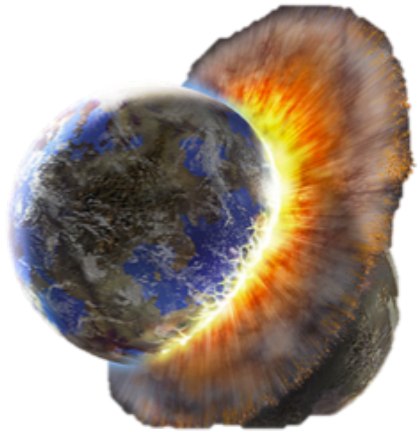
Virtual Reality & Physically-Based Simulation

Collision Detection



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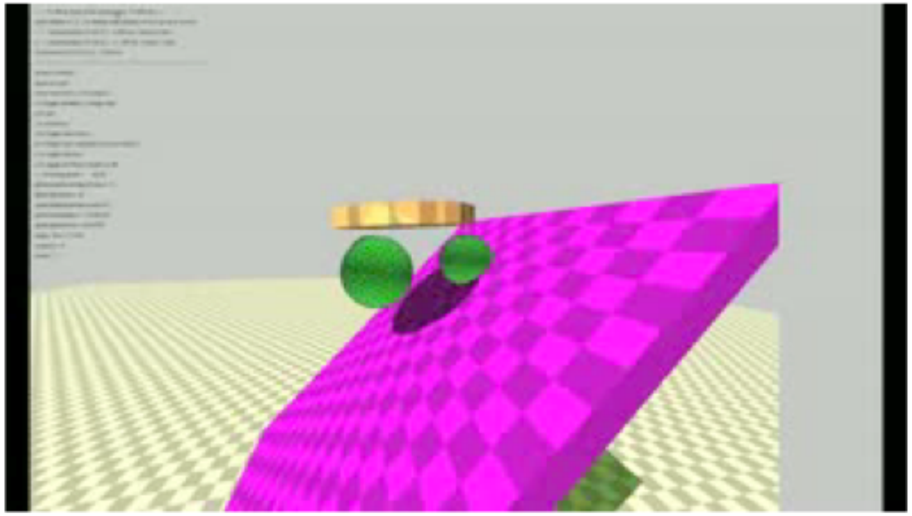
cgvr.cs.uni-bremen.de

Examples of Applications

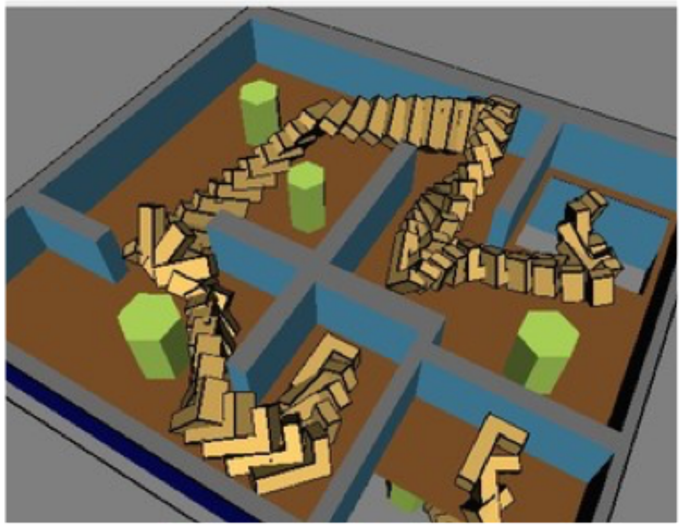
Virtual Prototyping

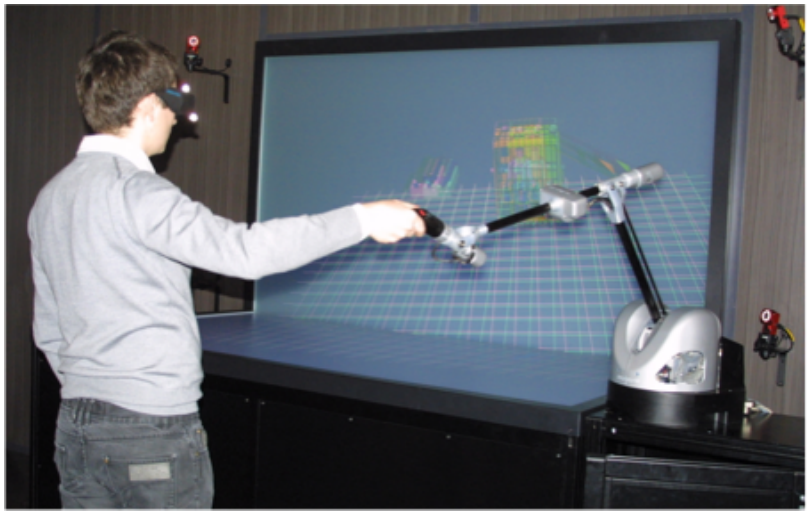


Physically-based simulation

Other Uses of Collision Detection

- Robotics: path planning (piano mover's problem)
- Medical training simulators
- Rendering of force feedback





Collision Detection Within Simulations

- Main loop:

  Move objects

  Check collisions

  Handle collisions (e.g., compute penalty forces)

- Collisions pose two different problems:
  1. Collision detection
  2. Collision handling (e.g., physically-based simulation, or visualization)

- In this chapter: only collision detection

Definitions

- Given $P, Q \subseteq \mathbb{R}^3$
- The detection problem:

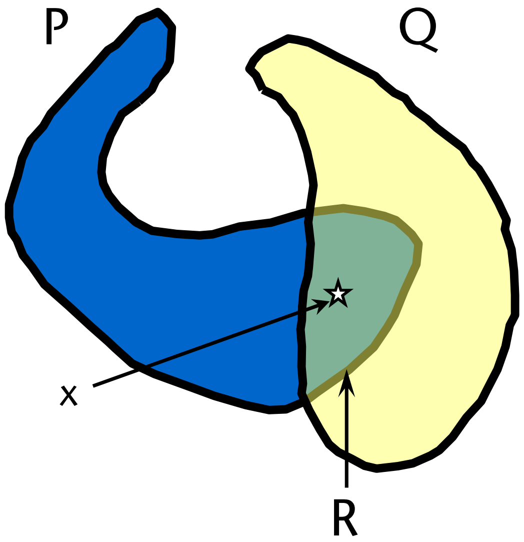
  "P and Q collide" $:\Leftrightarrow$

  $P \cap Q \neq \emptyset \Leftrightarrow$

  $\exists x \in^3 : x \in P \wedge x \in Q$

- The construction problem:

  compute $R := P \cap Q$



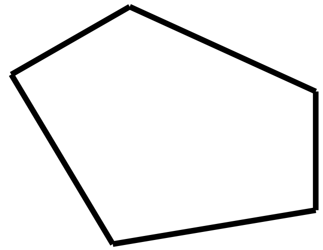
- For polygonal objects we define collisions as follows:

  P,Q collide $\Leftrightarrow \quad \exists f \in F^P \exists f' \in F^Q : f \cap f' \neq \emptyset$

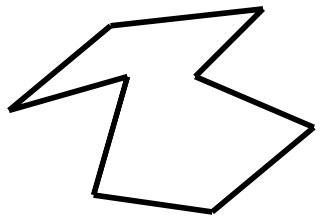
- The games community often has a different definition of "collision"

Classes of Objects

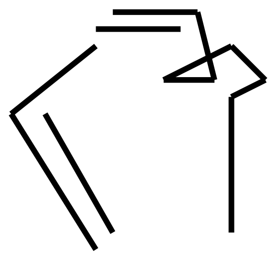
- Convex
- Closed and simple
  (no self-penetrations)
- Polygon soups
  - Not necessarily closed
  - Duplicate polygons
  - Coplanar polygons
  - Self-penetrations
  - Degenerate cardigans
  - Holes
- Deformable



Convex



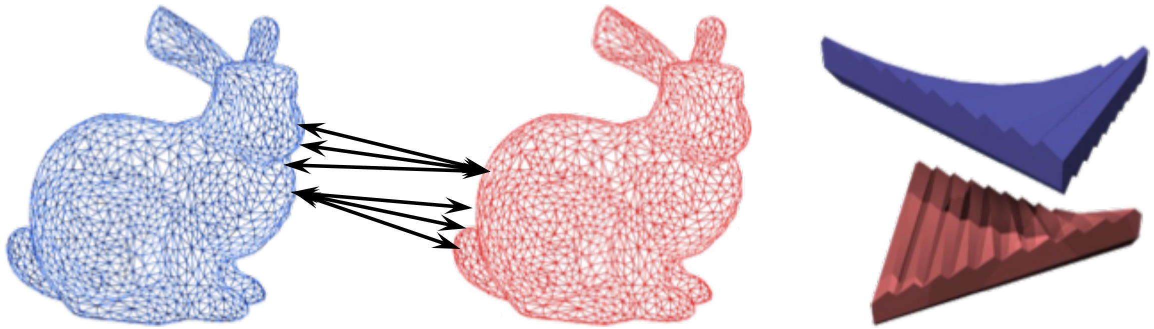
Simple & closed



Polygon soup

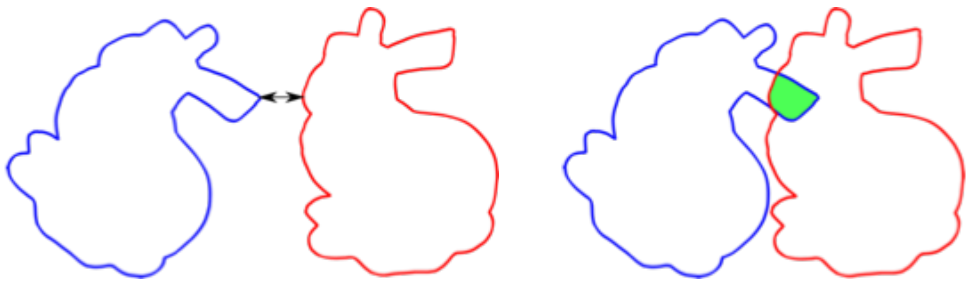
Why is Collision Detection so Hard?

1. All-pairs weakness:



2. Discrete time steps:

3. Efficient computation of proximity / penetration:

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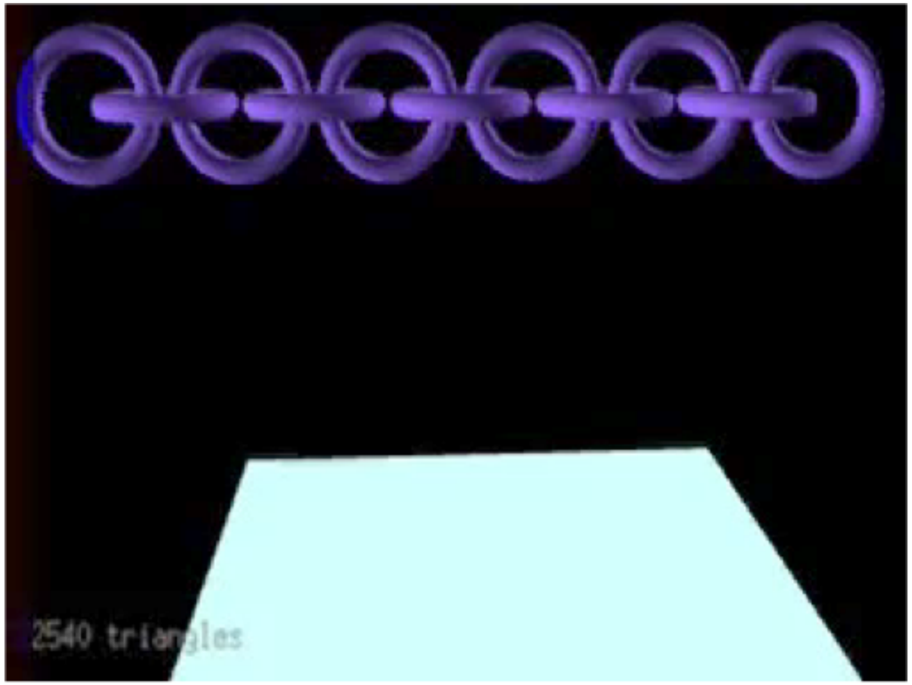
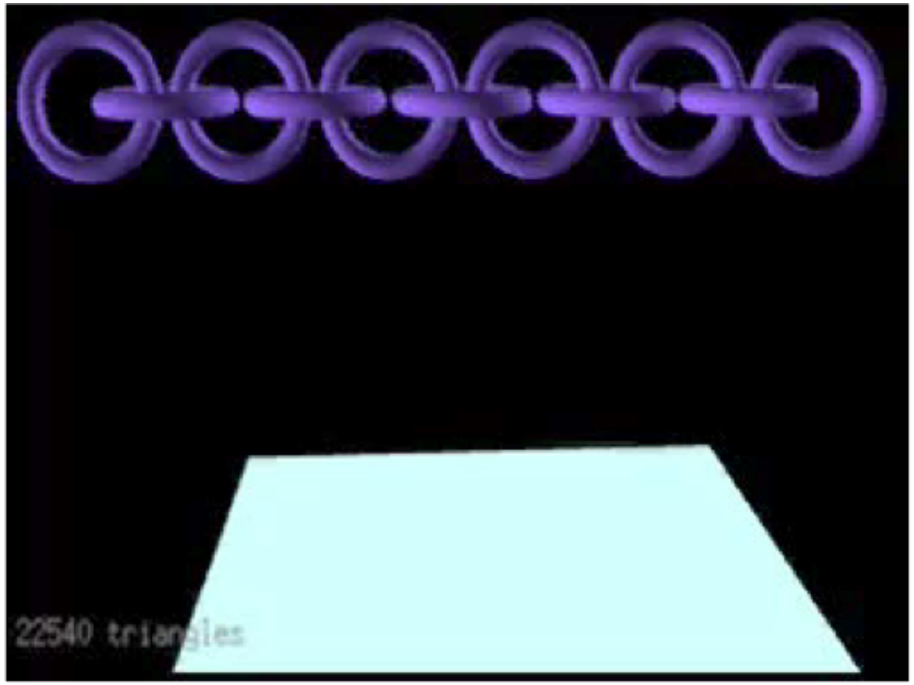
Importance of the Performance of Collision Detection



naïve algorithm
(test all pairs of polygons)



clever algorithm
(use bbox hierarchy)

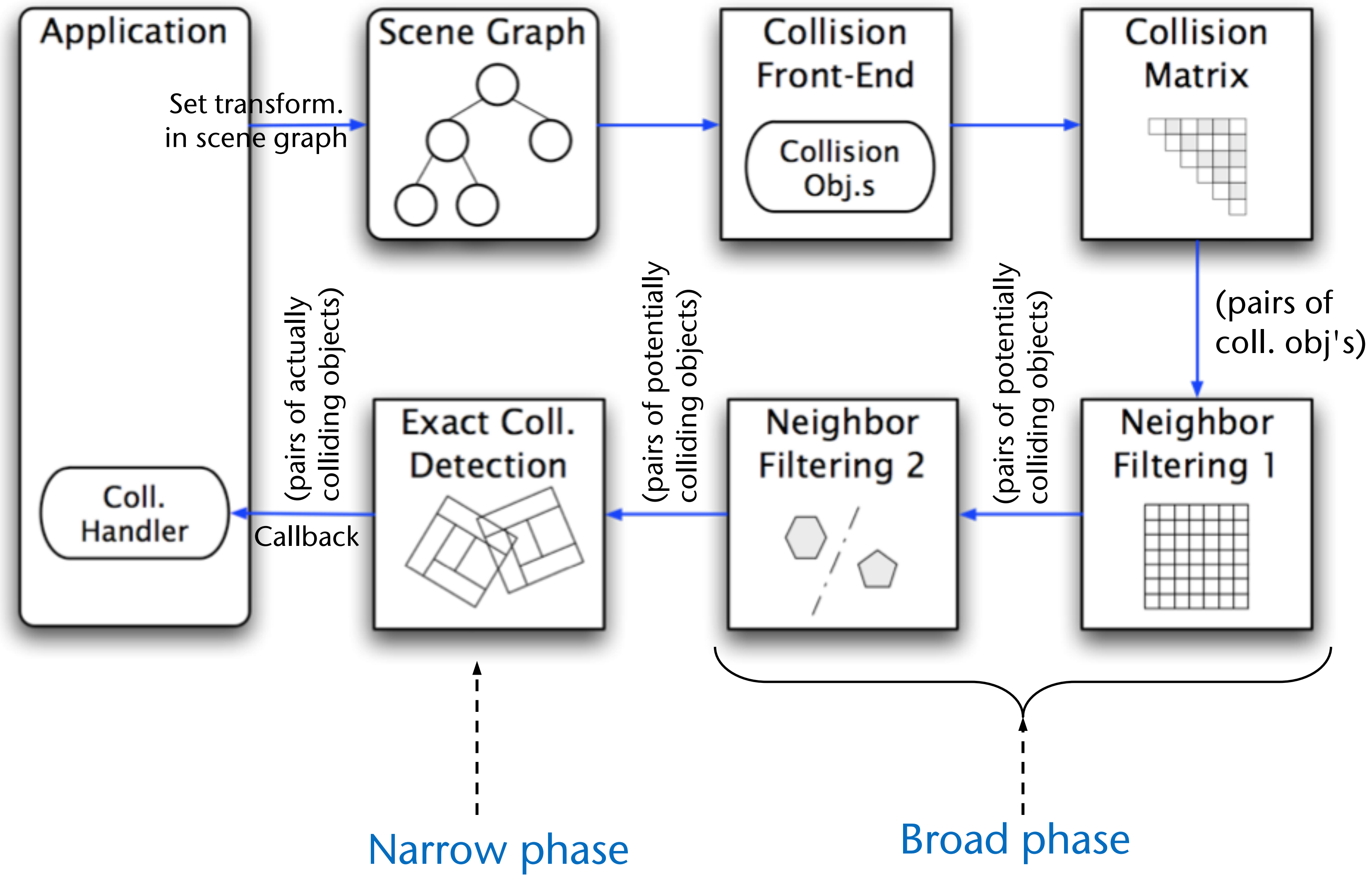
Conclusion: the performance of the algorithm for collision detection determines (often) the overall performance of the simulation!

In many simulations, the coll.det. part takes 60-90 % of the overall time

Requirements on Collision Detection

- Handle a large class of objects
- Lots of moving objects (1000s in some cases)
- Very high performance, so that a physically-based simulation can do many iterations per frame (at least 2x 100,000 polygons in <1 millisec)
- Return a contact point ("witness") in case of collision
  - Optionally: return *all* intersection points
- Auxiliary data structures should not be too large (<2x memory usage of originial data)
  - Preprocessing for these auxiliary data structures should not take too long, so that it can be done at startup time (< 5sec / object)

The Collision Detection Pipeline



The Collision Interest Matrix

- Interest in collisions is specific to different applications/modules:
  - Not all modules in an application are interested in all possible collisions;
  - Some pairs of objects collide all the time, some can never collide;
- Goal: prevent unnecessary collision tests

  ⇒ Collision Interest Matrix

Obj	1	2	3	4	5	6	7	8
1			x	x	x	x		
2						x		
3						x		x
4								x
5								x
6								x
7								x
8								

- The elements in this matrix comprise:
  - Flag for collision detection
  - Additional info that needs to be stored from frame to frame for each pair for certain algorithms (e.g., the separating plane)
  - *Callbacks* in die Module

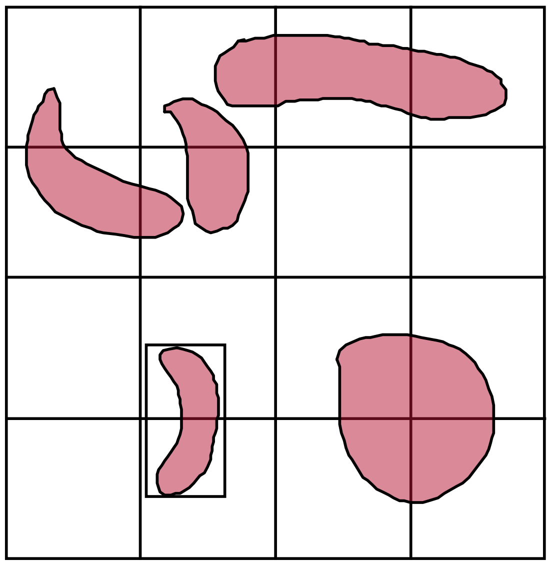
Methods for the Broad Phase

- Broad phase = one or more filtering step
  - Goal: quickly filter pairs of objects that cannot intersect because they are *too far away* from each other
- Standard approach:
  - Enclose each object within a bounding box (bbox)
  - Compare the 2 bboxes for a given pair of objects
- Assumption: n objects are moving
- ➢ *Brute-force* method needs to compare $\mathcal{O}(n^2)$ bboxes
- Goal: determine neighbors more efficiently
- ➢ 3D grid, sweep plane techniques ("sweep and prune"), feature tracking on convex hulls, etc.

The 3D Grid

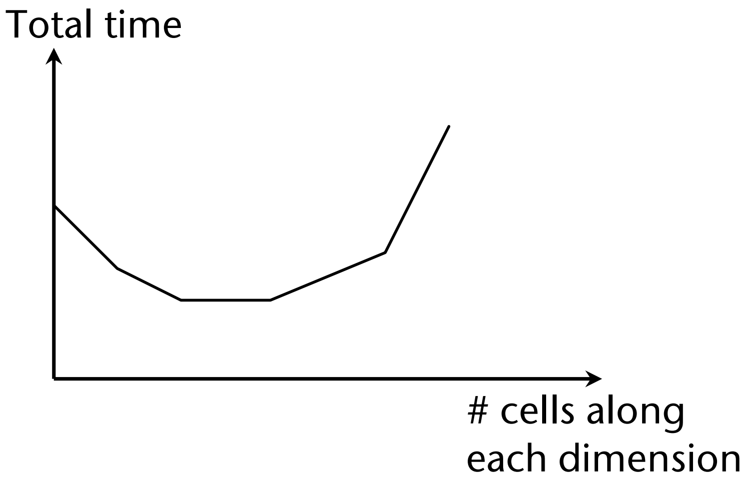
1. Partition the "universe" by a 3D grid
2. Objects are considered neighbors, if they occupy the same cell
3. Determine cell occupancy by bbox
4. When objects move → update grid

- Neighbor-finding = find all cells that contain more than one obj
  - Data structure here: hash table (!)
  - Collision in hash table → probably neighbor



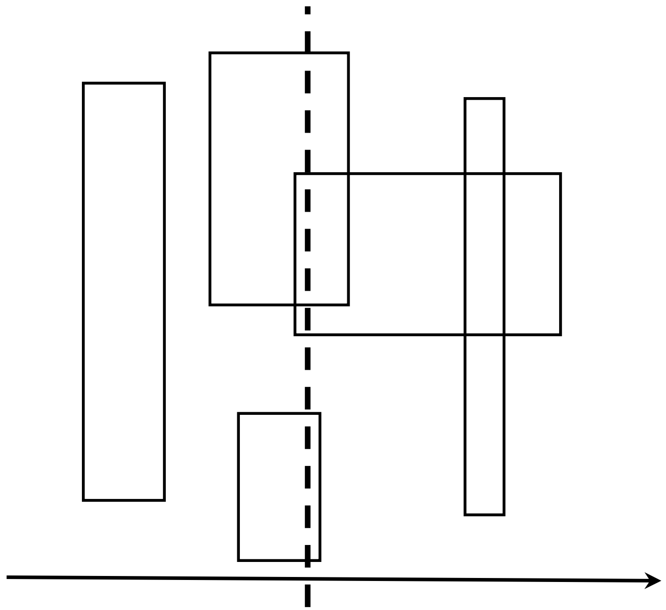
The trade-off:

- Fewer cells = larger cells
  - Distant objects are still "neighbors"
- More cells = smaller cells
  - Objects occupy more cells
  - Effort for updating increases
- Rule of thumb: cell size ≈ avg obj diameter



The Plane Sweep Technique (aka Sweep and Prune)

- The idea:
  sweep plane through space
  perpendicular to the X axis
- The algorithm:



```
sort the X coordinates of all boxes
start with the leftmost box
keep a list of active boxes
loop over x-coords (= left/right box borders):
  if current box border is the left side (= "opening"):
     check this box against all boxes in the active list
     add this box to the list of active boxes
  else (= "closing"):
     remove this box from the list of active boxes
```

Temporal Coherence

- Observation:

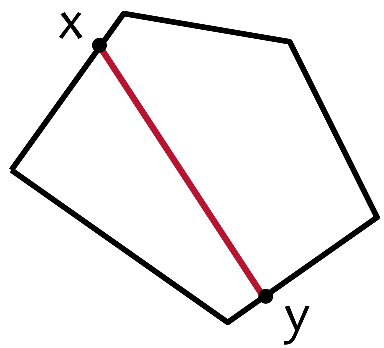
  Two consecutive images in a sequence differ only by very little (usually).

- Terminology: temporal coherence (a.k.a. frame-to-frame coherence)
- Examples:
  - Motion of a camera
  - Motion of objects in a film / animation
- Applications:
  - Computer Vision (e.g. tracking of markers)
  - MPEG
  - Collision detection
  - Ray-tracing of animations (e.g. using kinetic data structures)
- Algorithms based on frame-to-frame coherence are called "incremental", sometimes "dynamic" or "online" (albeit the latter is the wrong term)

Convex Objects

- Definition of "convex polyhedron":

$$P \subset \mathbb{R}^3 \quad \text{convex} \Leftrightarrow$$
$$\forall x, y \in P : \overline{xy} \subset P \Leftrightarrow$$
$$P = \bigcap_{i=1\ldots n} H_i \quad , H_i = \text{half-spaces}$$

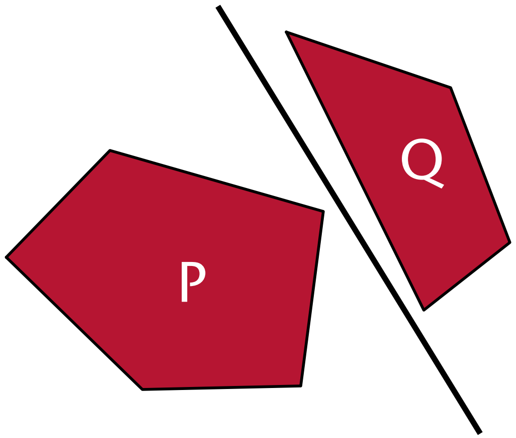


- A condition for "non-collision":

  P and Q are "linearly separable" $:\Leftrightarrow$

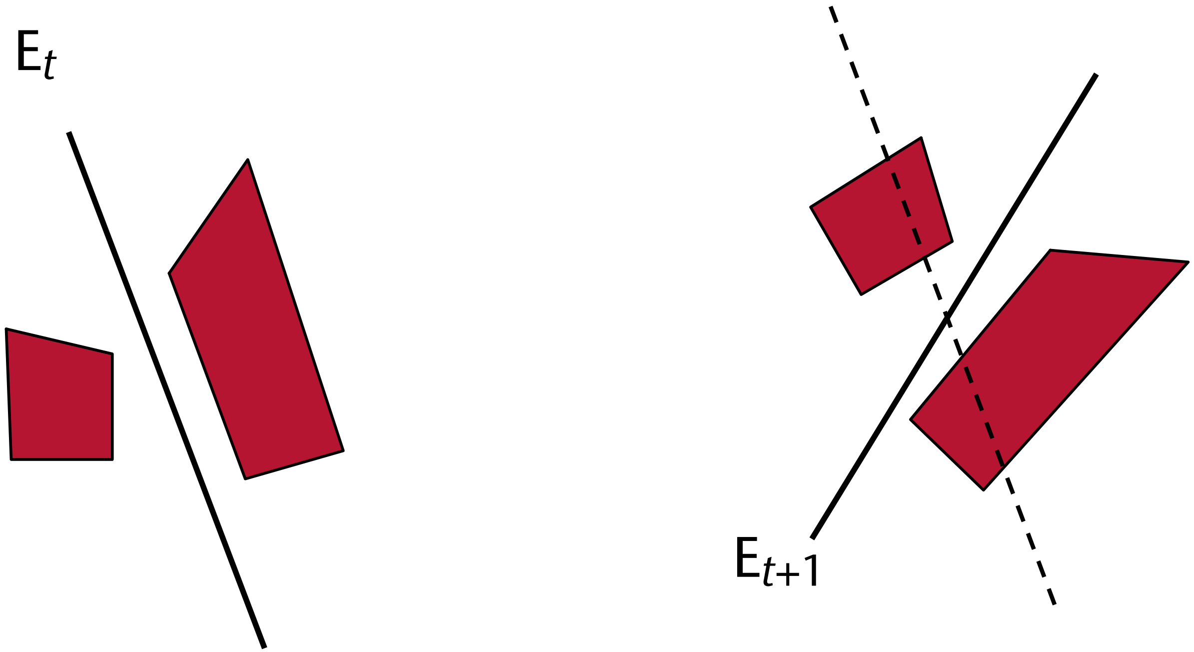
$$\exists \text{ half-space } H : P \subseteq H \wedge Q \subseteq H^c$$

("P is completely on one side of H,
Q completely on the other side")



The "Separating Planes" Algorithm

- The idea: utilize temporal coherence →
  if E_t was a separating plane between P and Q at time t, then the new separating plane E_{t+1} is probably not very "far" from E_t (perhaps it is even the same)



load E_t = separating plane between P & Q at time t

$E := E_t$

repeat max n times

  if exists $v \in \text{vertices}(P)$ on the back side of E:

    rot./transl. E such that v is now on the front side of E

  if exists $v \in \text{vertices}(Q)$ on the front side of E:

    rot./transl. E such that v is now on the back side of E

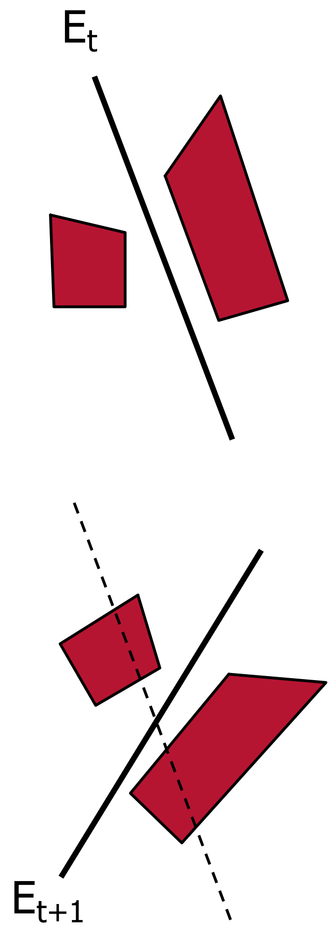
  if there are no vertices on the "wrong" side of E, resp.:

    return "no collision"

if there are still vertices on the "wrong" side of E:

  return "collision" {could be wrong}

save $E_{t+1} := E$ for the next frame



For details on the "rot./transl. E" step → see perceptron learning algorithm

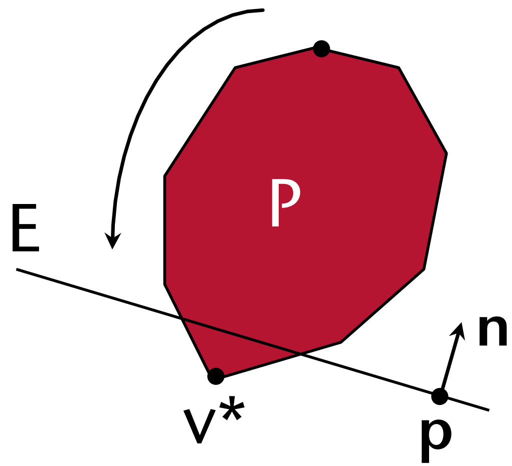
How to Find a Vertex on the "Wrong" Side *Quickly*

- The brute-force method:

  test all **v** whether $f(\mathbf{v}) = (\mathbf{v} - \mathbf{p}) \cdot \mathbf{n} > 0$

- Observation:

  1. *f* is linear,
  2. P is convex $\Rightarrow$ *f*(*x*) has (usually) exactly one minimum over all points *x* on the surface of P
  3. $\exists^1 \mathbf{v}^* \; : \; f(\mathbf{v}^*) = \min$

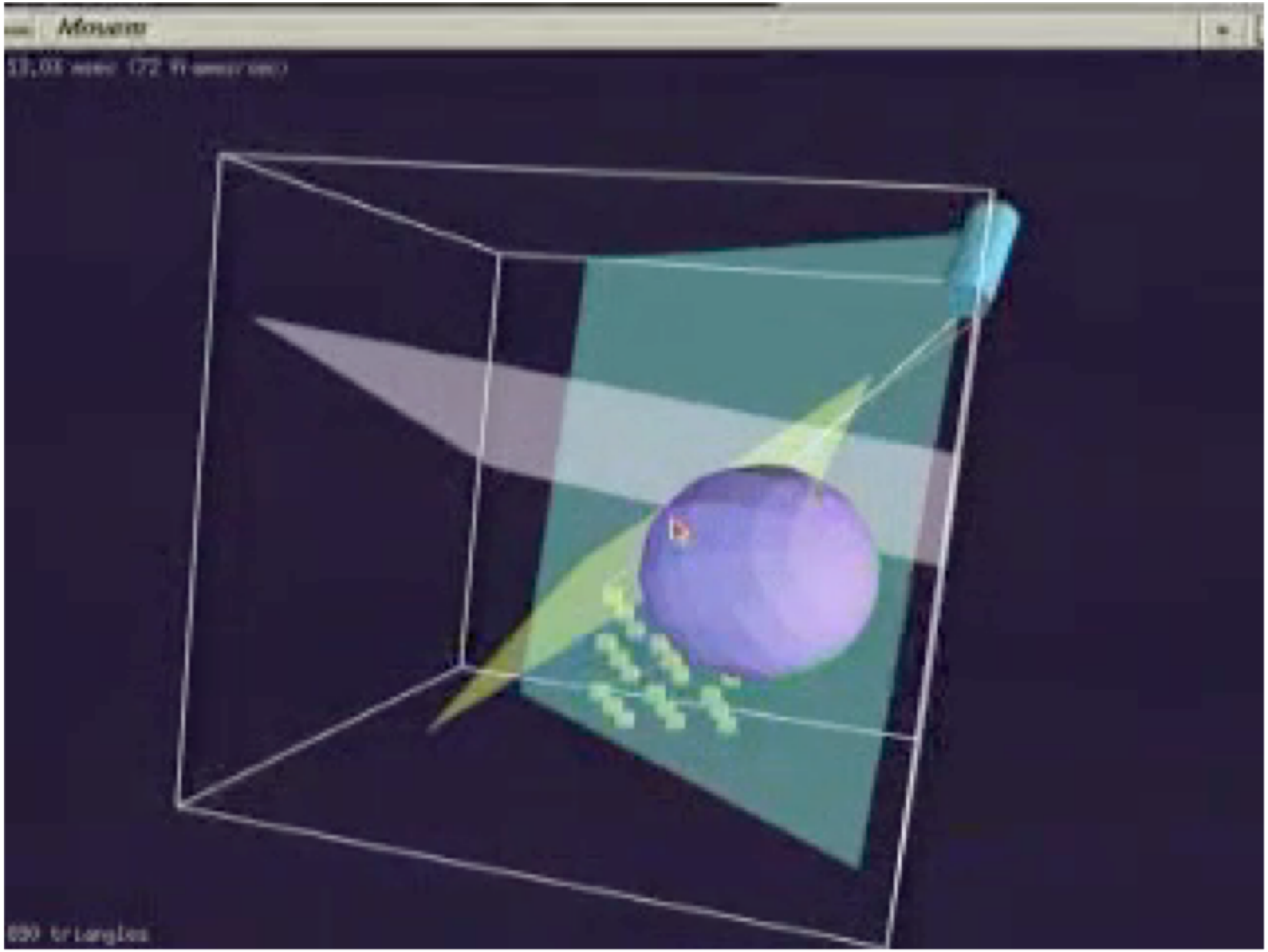


- The algorithm (steepest descent on the surface w.r.t. *f*):
  - Start with an arbitrary vertex **v**
  - Walk to the neighbor **v'** of **v** for which $f(\mathbf{v}') = \min.$ (among all neighbors)
  - Stop if there is no neighbor **v'** of **v** for which $f(\mathbf{v}') < f(\mathbf{v})$

Properties of this Algorithm

- + Expected running time is in O(1)!
  The algo exploits *frame-to-frame coherence*:
  if the objects move only very little, then the algo just checks whether the old separating plane is still a separating plane;
  if the separating plane has to be moved, then the algo is often finished after a few iterations.
- + Works even for deformable objects, so long as they stay convex
- – Works only for convex objects
- – Could return the wrong answer if P and Q are extremely close but not intersecting (bias)
- *Research question: can you find an un-biased (deterministic) variant?*

Visualization



Closest Feature Tracking Optional

- Idea:
  - Maintain the minimal distance between a pair of objects
  - Which is realized by one point on the surface of each object
  - If the objects move continuously, then those points move continuously on the surface of their objects
- The algorithm is based on the following methods:
  - Voronoi diagrams
  - The “closest features” lemma

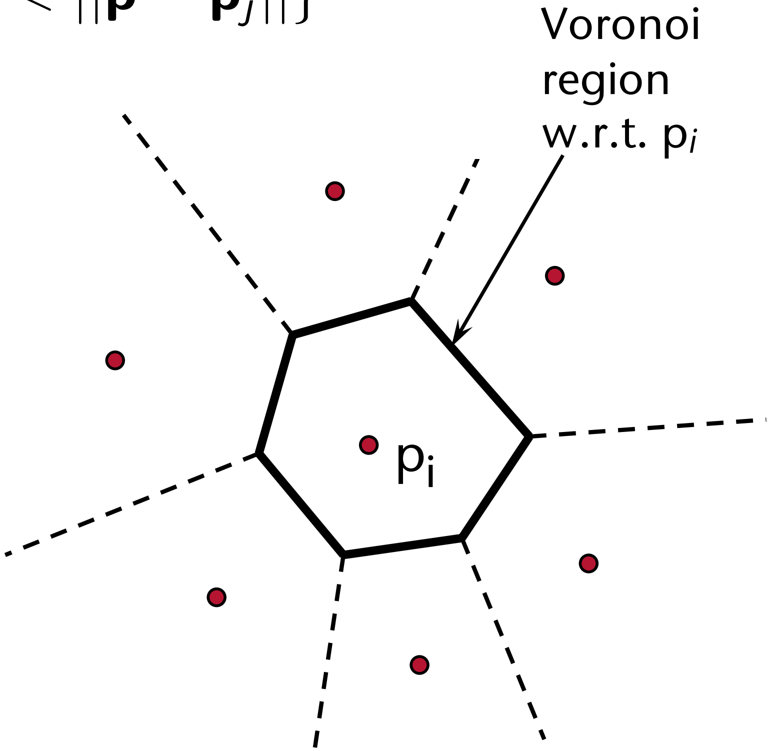
Voronoi Diagrams for Point Sets

Optional

- Given a set of points $S = \{\mathbf{p}_i\}$, called sites (or generators)
- Definition of a Voronoi region/cell :

$$V(p_i) := \{\mathbf{p} \in \mathbb{R}^2 \mid \forall j \neq i : ||\mathbf{p} - \mathbf{p}_i|| < ||\mathbf{p} - \mathbf{p}_j||\}$$

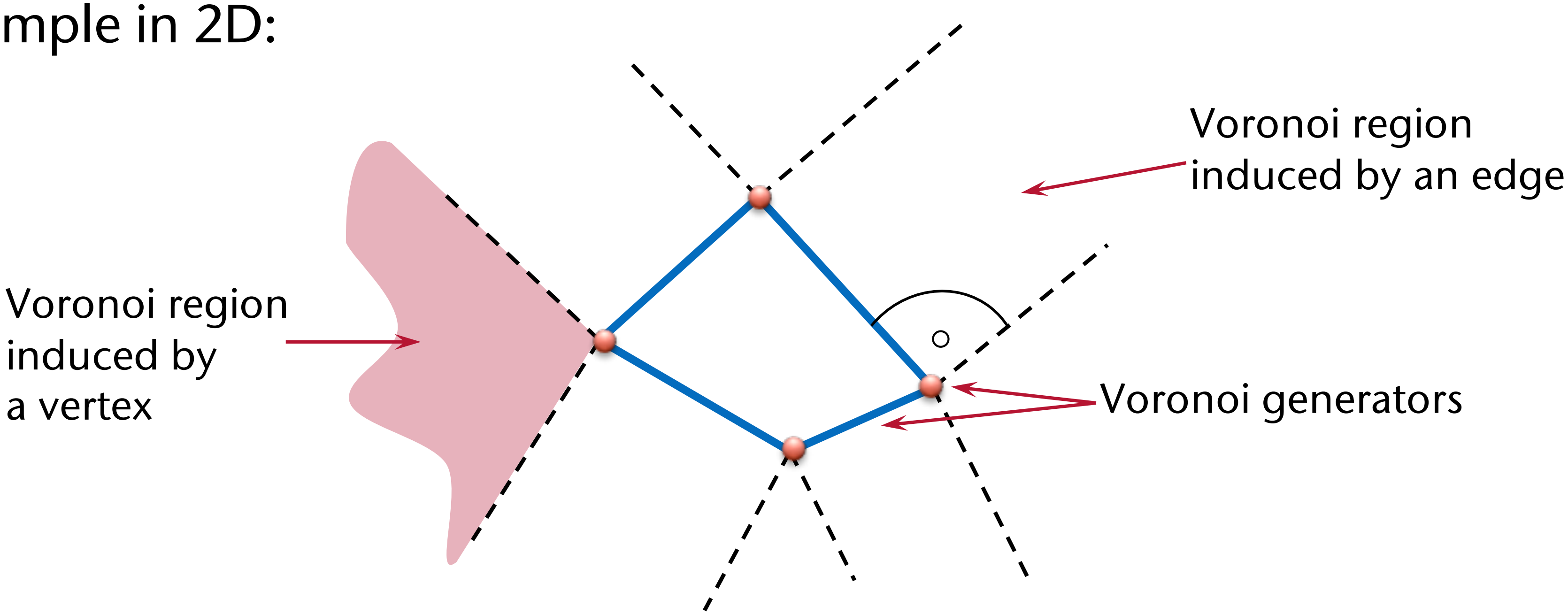
- Definition of Voronoi diagrams:
  The Voronoi diagram $\mathcal{VD}(S)$ over a set of points S is the union of all Voronoi regions over the points in S.
- $\mathcal{VD}(S)$ induces a partition of the plane into Voronoi edges, Voronoi nodes, and Voronoi regions



- Interaktive Demo: http://web.cs.uni-bonn.de/I/GeomLab/VoroGlide/

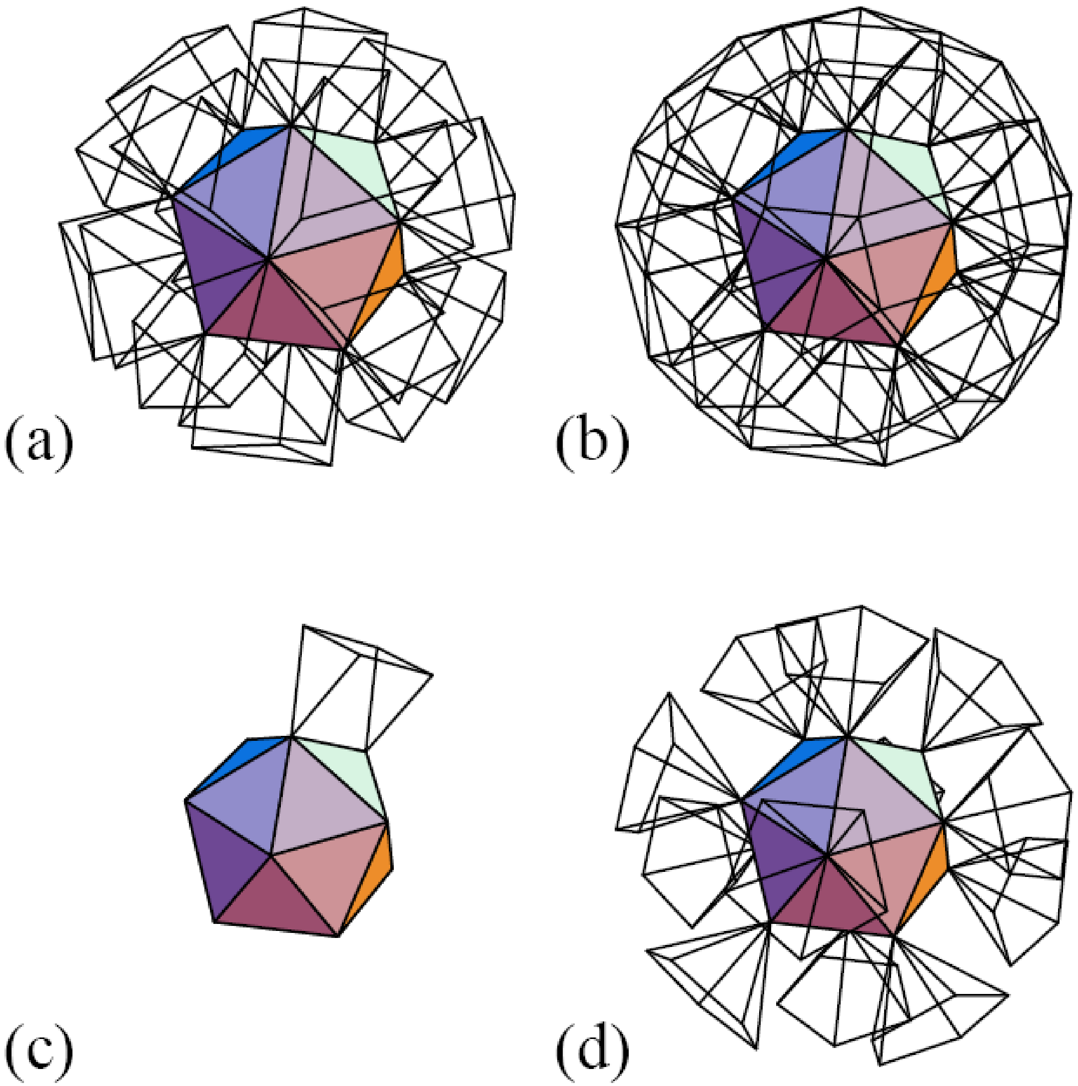
Voronoi Diagrams over Sets of Points, Edges, Polygons

- Voronoi diagrams can be defined analogously in 3D (and higher dimensions)
- What if the generators are not points but edges / polygons?
- Definition of a Voronoi cell is still the same:
  The Voronoi region of an edge/polygon := all points in space that are closer to "their" generator than to any other
- Example in 2D:



Outer Voronoi Regions Generated by a Polyhedron

Optional



The external Voronoi regions of …

(a) faces

(b) edges

(c) a single edge

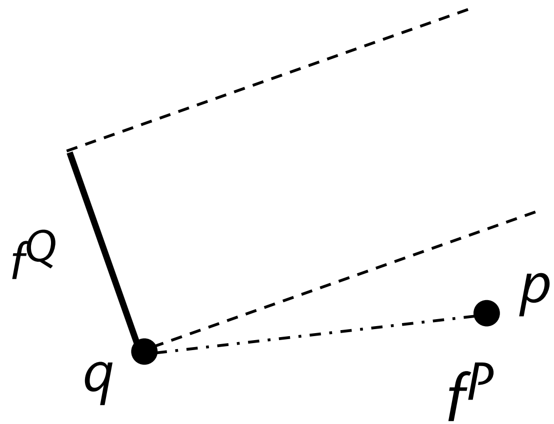
(d) vertices

Outer Voronoi regions for convex polyhedra can be constructed very easily!
(We won't need inner Voronoi regions.)

Closest Features

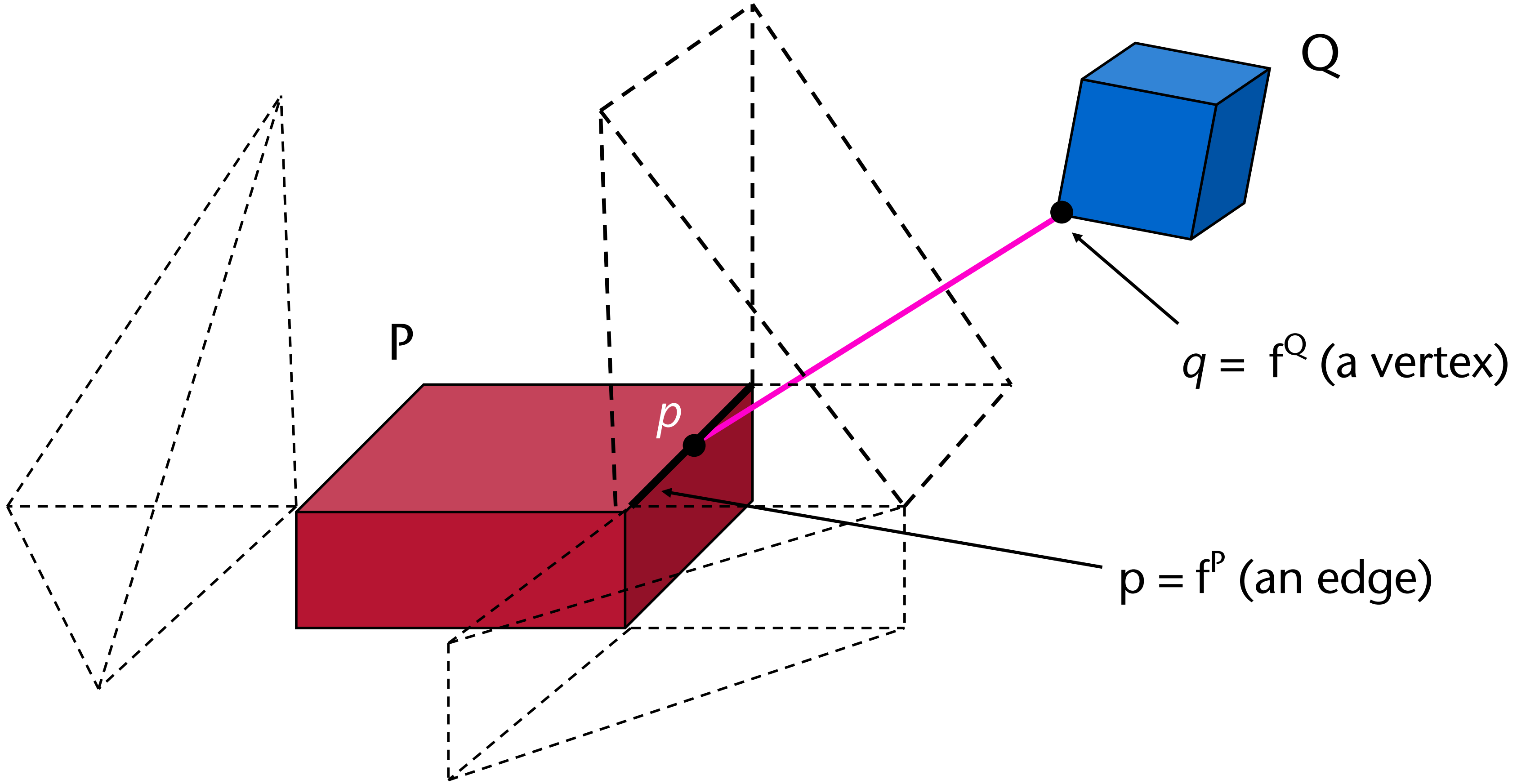
Optional

- Definition *Feature* f^P := a vertex, edge, polygon of polyhedron P.
- Definition "Closest Feature":
  Let f^P and f^Q be two features on polyhedra P and Q, resp., and let p, q be points on f^P and f^Q, resp., that realize the minimal distance between P and Q, i.e.
  $$d(P,Q) = d\left(f^P, f^Q\right) = ||p-q||$$
  Then f^P and f^Q are called "closest features".
- The "closest feature" lemma:
  Let $V(f)$ denote the Voronoi region generated by feature f; let p and q be points on the surface of P and Q realizing the minimal distance. Then
  $$f^P, f^Q \text{ are closest features} \Leftrightarrow p \text{ is in } V(f^Q),\; q \text{ is in } V(f^P).$$



Example

Optional



The Algorithm (Another Kind of a Steepest Descent)

Optional

Start with two arbitrary features f^P, f^Q on P and Q, resp.

while (f^P, f^Q) are not (yet) closest features and dist(f^P, f^Q) > 0 :

  if (f^P,f^Q) has been considered already:

    return “collision” (b/c we've hit a cycle)

  compute p and q that realize the distance between f^P and f^Q

  if $p \in V(q)$ und $q \in V(p)$:

    return “no collision”, (f^P,f^Q) are the closest features

  if p lies on the "wrong" side of $V(q)$:

    f^P := the feature on that "other side" of $V(q)$

  do the same for q, if $q \notin V(p)$

if dist(f^P, f^Q) > 0 :

  return "no collision"

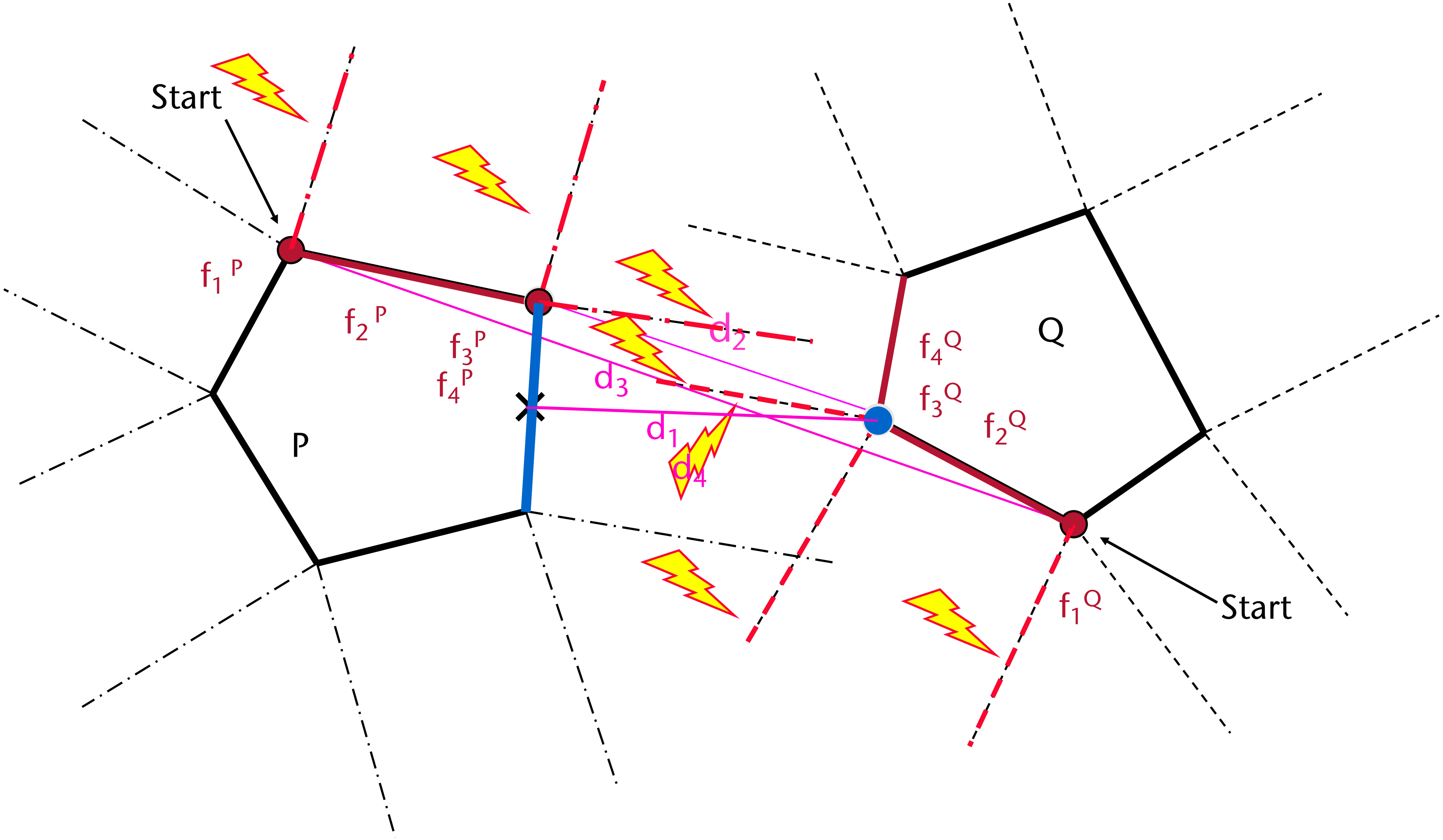
else

  return "collision"

Notice: in case of collision, some features are inside the other object, but we did not compute Voronoi regions inside objects!
→ hence the chance for cycles

Animation of the Algorithm

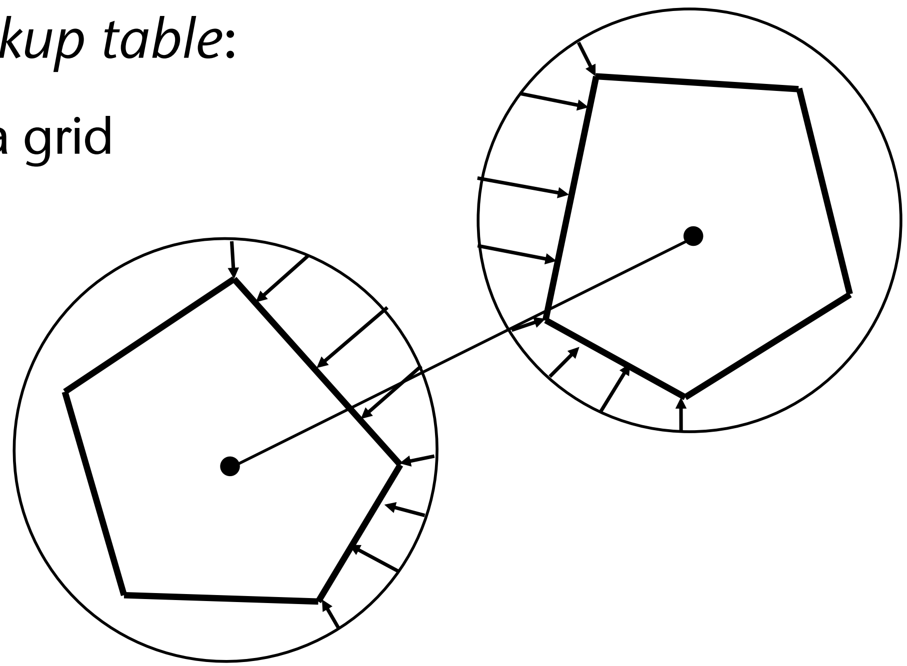
Optional



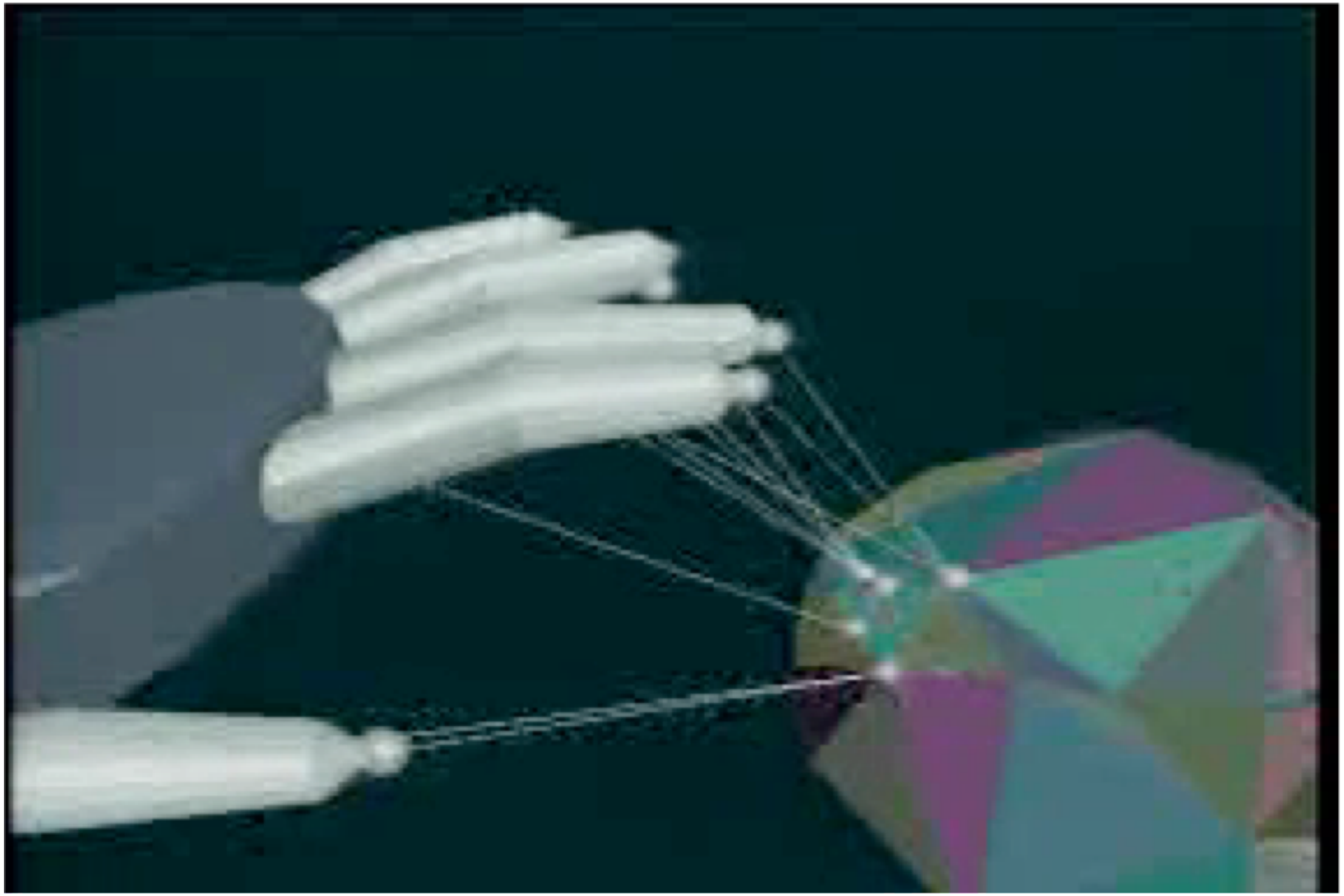
Some Remarks

Optional

- A little question to make you think:
  Actually, we don't really need the *Voronoi diagram!*
  (but with a *Voronoi diagram*, the algorithm is faster)
- The running time (in each frame) depends on the "degree" of temporal coherence
- Better initialization by using a *lookup table*:
  - Partition a surrounding sphere by a grid
  - Put each feature in each grid cell that it covers when propjected onto the sphere
  - Connect the two centers of a pair of objets by a line segment
  - Initialize the algorithm by the features hit by that line



Movie



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The Minkowski Sum

Optional

- Hermann Minkowski (1864 – 1909), German mathematician and physicist
- Definition (Minkowski Sum):
  Let A and B be subsets of a vector space;
  the Minkowski sum of A and B is defined as

$$A \oplus B = \{ \mathbf{a} + \mathbf{b} \,|\, \mathbf{a} \in A,\ \mathbf{b} \in B \}$$

- Analogously, we define the Minkowski difference:

$$A \ominus B = \{ \mathbf{a} - \mathbf{b} \,|\, \mathbf{a} \in A,\ \mathbf{b} \in B \}$$

- Clearly, the connection between Minkowski sum and difference:

$$A \ominus B = A \oplus (-B)$$

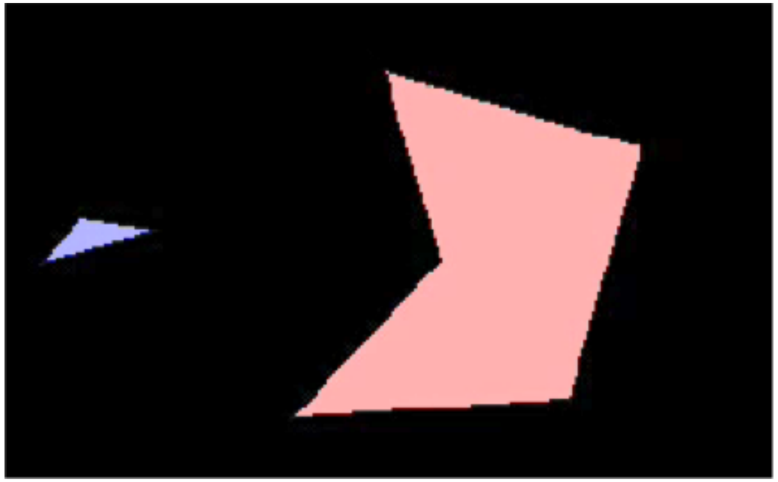
- Applications: computer graphics, computer vision, linear optimization, path planning in robotics, …

Some Simple Properties

Optional

- Commutative: $A \oplus B = B \oplus A$
- Associative: $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
- Distributive w.r.t. set union: $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
- Invariant against translation: $T(A) \oplus B = T(A \oplus B)$

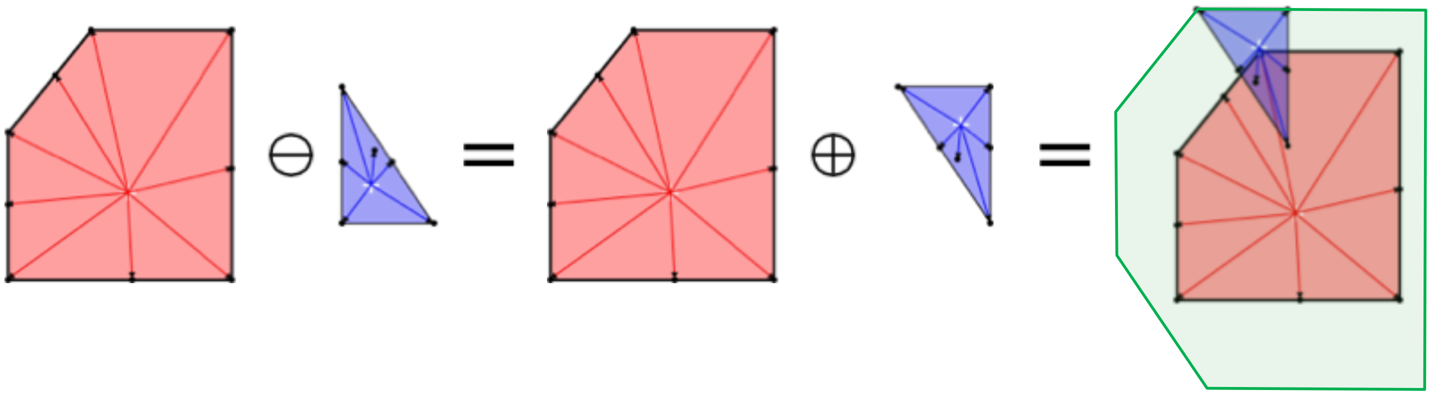
- Intuitive "computation" of the Minkowski sum/difference:



Warning: the yellow polygon in the animation shows the Minkowsi sum **modulo**(!) possible translations!

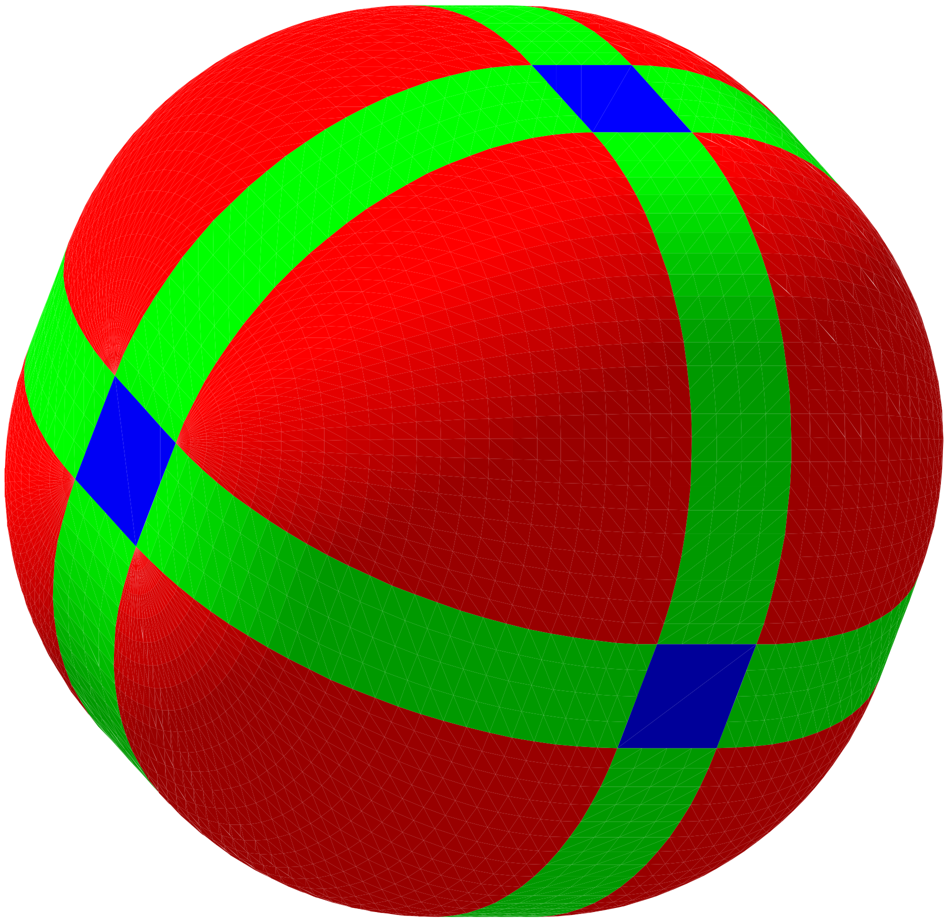
- Analogous construction of Minkowski difference:

$$A \ominus B = A \oplus -B = C$$



Visualizations of Simple Examples

Optional



Minkowski sum of a ball and a cube

Minkowski sum of cube and cone, only the cone is rotating

Minkowski sum of cube and cone, both are translating

Optional

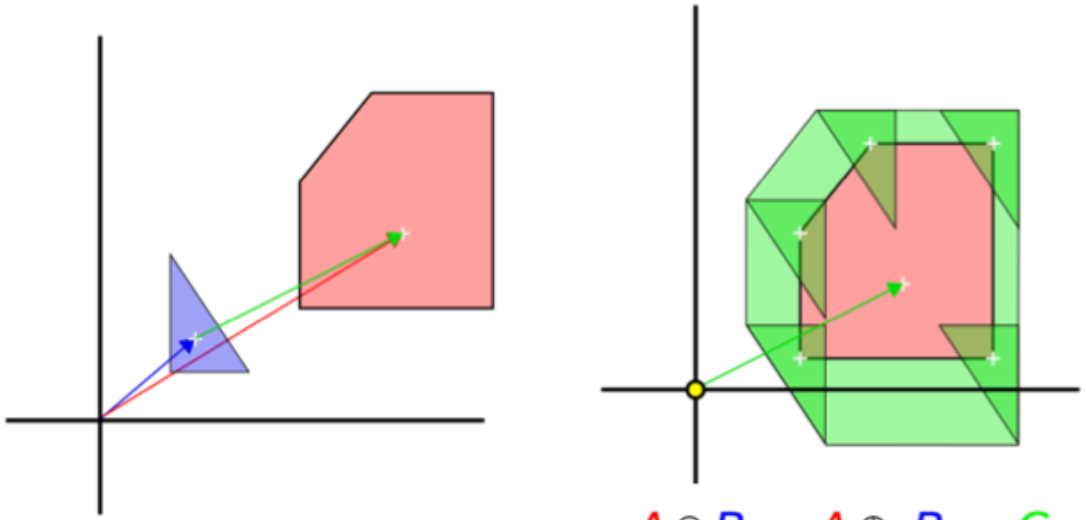
The Complexity of the Minkowski Sum (in 2D, without proofs)

- Let A and B be polygons with n and m vertices, resp.:
  - If both A and B are convex,then $A \oplus B$ is convex, too, and has complexity $O(m+n)$
  - If only B is convex, then $A \oplus B$ has complexity $O(mn)$
  - If neither is convex, then $A \oplus B$ has complexity $O(m^2 n^2)$
- Algorithmic complexity of the computation of $A \oplus B$:
  - If A and B are convex, then $A \oplus B$ can be computed in time $O(m+n)$
  - If only B is convex, then $A \oplus B$ can be computed in randomized time $O(mn\, log^2(mn))$
  - If neither is convex, then $A \oplus B$ can be computed in time $O(mn^2\, log(mn))$

An Intersection Test for Two Convex Objects using Minkowski Sums

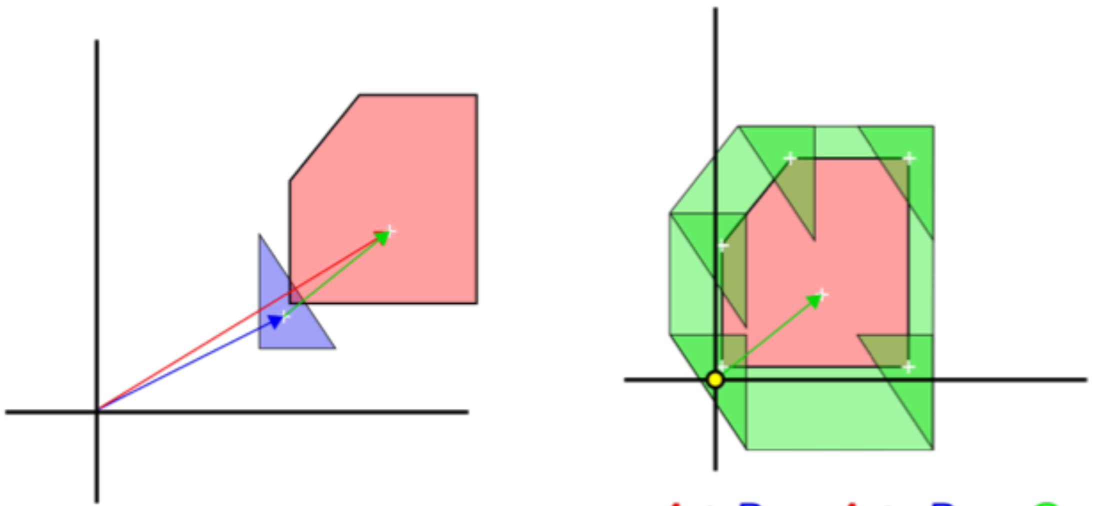
- Translate both objects so that the coordinate system's origin 0 is inside *B*
- Compute the Minkowski difference
- A and B intersect ⇔

$$0 \in A \ominus B$$



$A \ominus B = A \oplus -B = C$

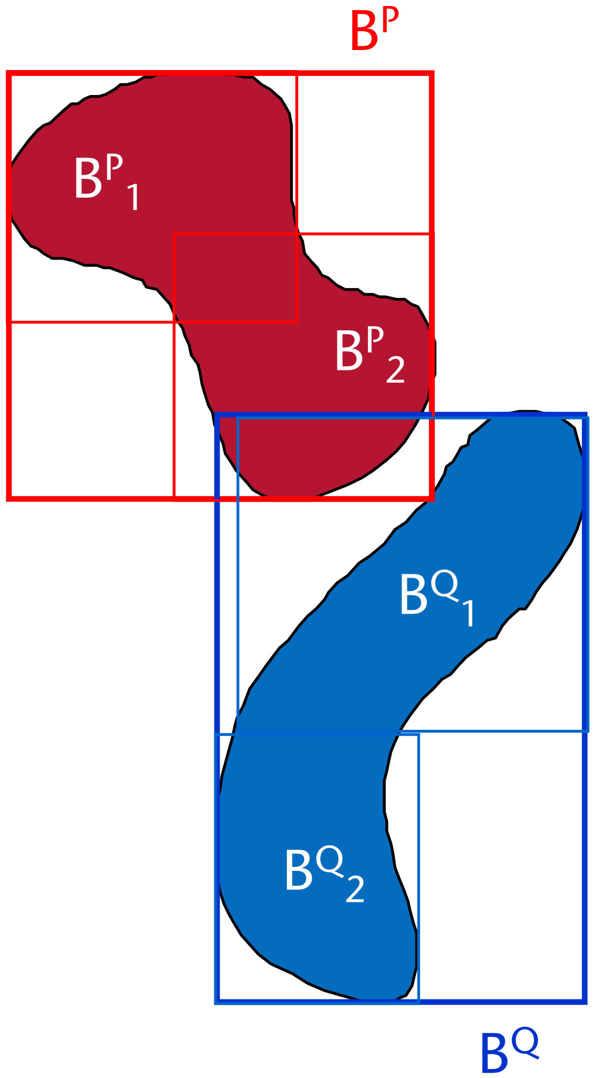
- Example where an intersection occurs:



$A \ominus B = A \oplus -B = C$

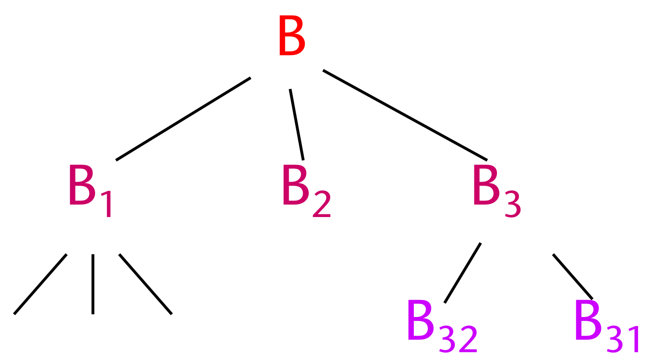
Hierarchical Collision Detection

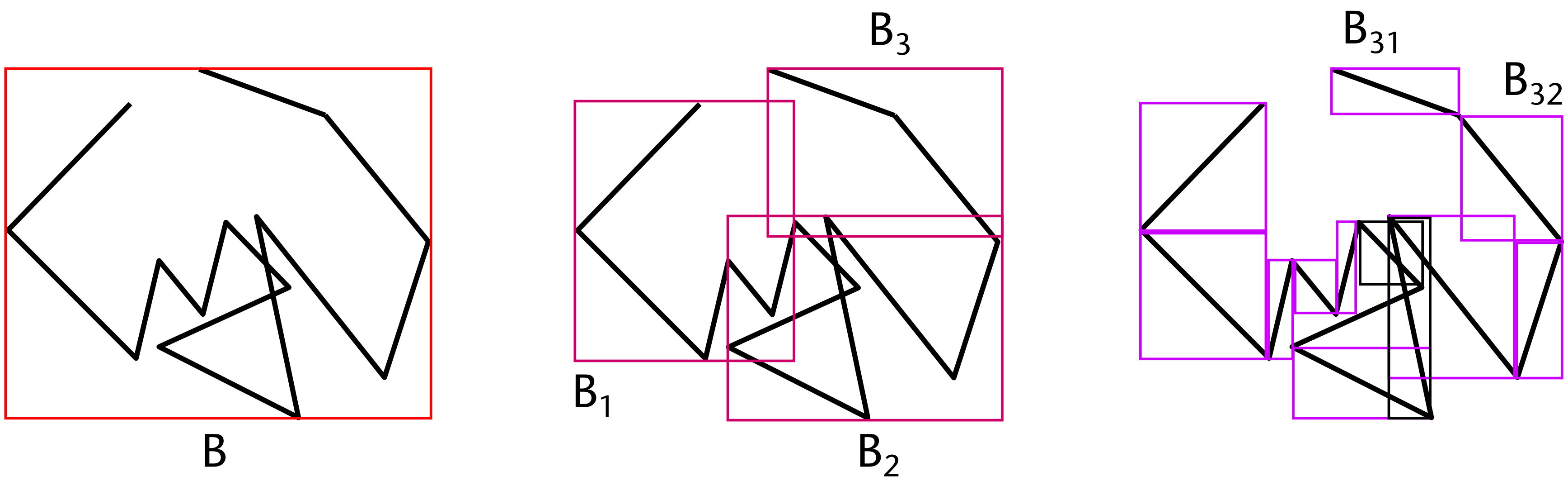
- *The* standard approach for "polygon soups"
- Algorithmic technique: divide & conquer



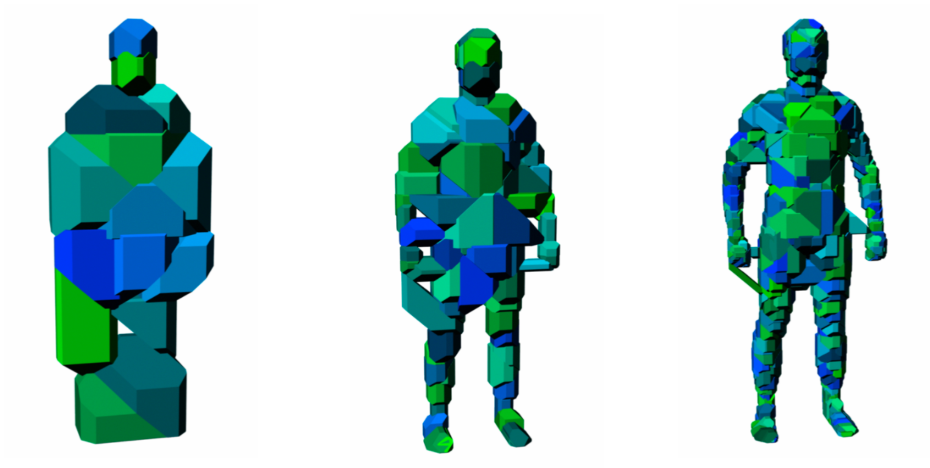
The Bounding Volume Hierarchy (BVH)

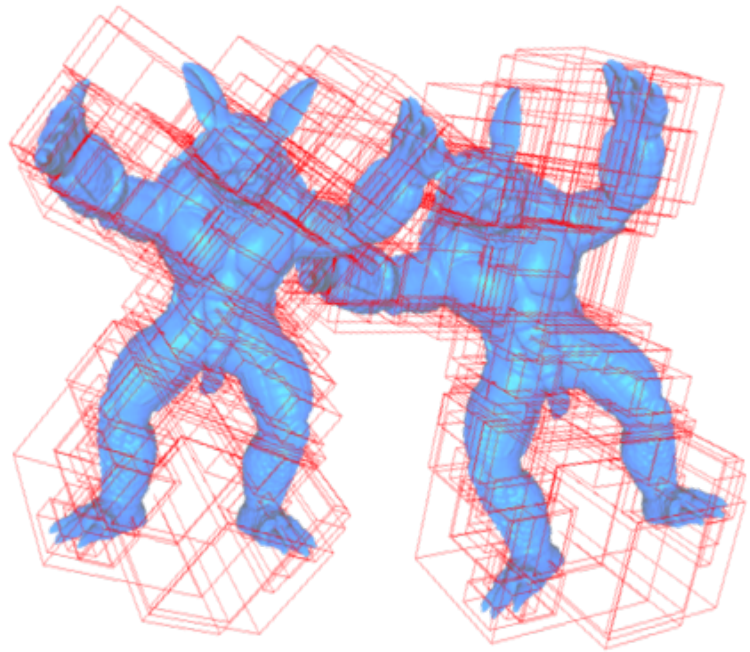
- Constructive defintion of a bounding volume hierarchy:
  1. Enclose all polygons, P, in a bounding volume BV(P)
  2. Partition P into subsets $P_1, \ldots, P_n$
  3. Rekursively construct a BVH for each P_i and put them as children of P in the tree
- Typical arity = 2 or 4

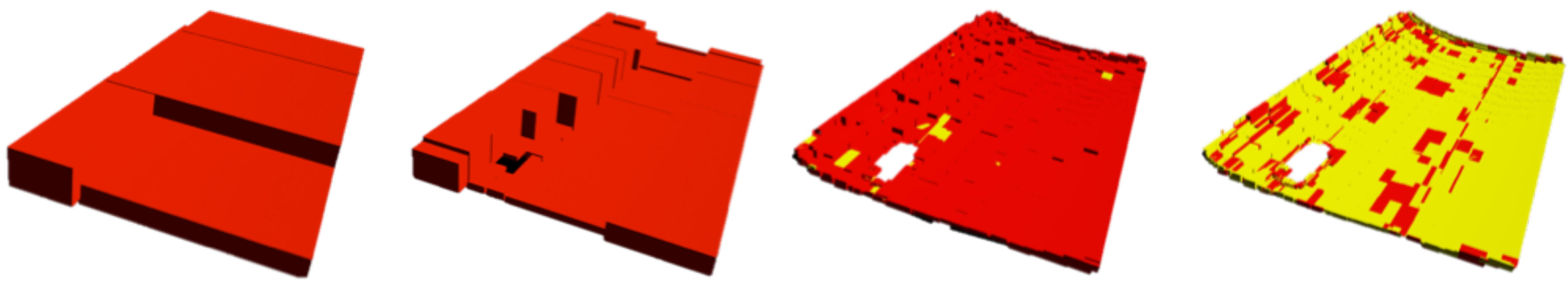




- Visualizations of different levels of some BVHs:



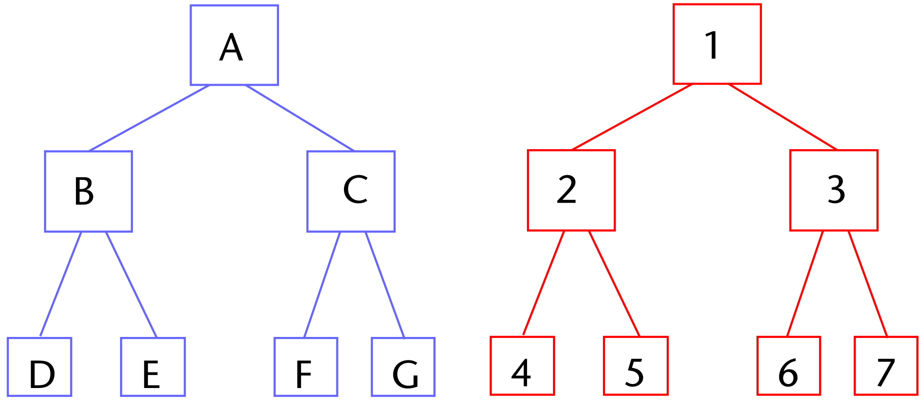


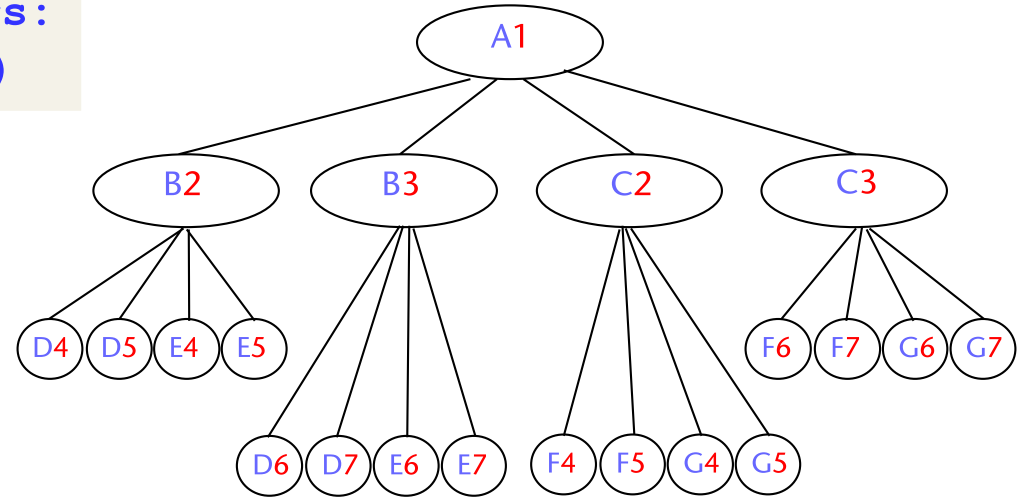


The General Hierarchical Collision Detection Algo

- Simultaneous traversal of two BVHs:

```
traverse( node X, node Y ):
if X,Y do not overlap:
    return
if X,Y are leaves:
    check polygons
else
    for all children pairs:
        traverse( Xi, Yj )
```



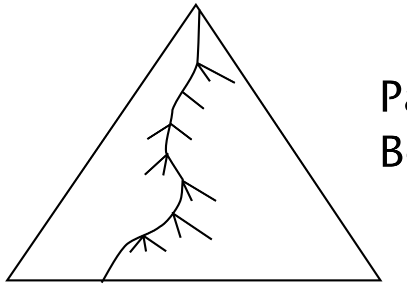


Resulting, conceptual(!) Bounding Volume Test Tree (BVTT)

A Simple Running Time Estimation

Optional

- *Best-case*: $O(\log n)$



Path through the
Bounding Volume Test Tree (BVTT)

- Extremely simple *average-case* estimation:
  - Let P[k] = probability that *exactly* k children pairs overlap, $k \in [0,\ldots,4]$

$$P[k] = \binom{4}{k} / 16 \, , \quad P[0] = \frac{1}{16}$$

  - Assumption: all events are equally likely, each subtree has ½ of the polygons
  - Expected running time:

$$T(n) = \tfrac{1}{16} \cdot 0 + \tfrac{4}{16} \cdot T(\tfrac{n}{2}) + \tfrac{6}{16} \cdot 2T(\tfrac{n}{2}) + \tfrac{4}{16} \cdot 3T(\tfrac{n}{2}) + \tfrac{1}{16} \cdot 4T(\tfrac{n}{2})$$

$$T(n) = 2T(\tfrac{n}{2}) \in O(n)$$

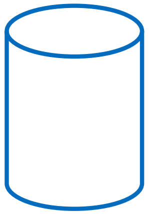
- In praxi: running time is better/worse depending on degree of overlap

Different Kinds of Bounding Volumes

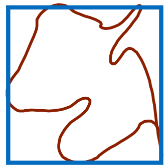
Requirements (for collision detection):

- *Very* fast overlap test → "simple" BVs
  - Even if BVs have been translated/rotated
- Little overlap among BVs on the same level in a BVH (i.e., if you want to cover the whole space with the BVs, there should be as little overlap as possible) → "*tight BVs*"

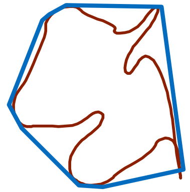
Different Kinds of Bounding Volumes



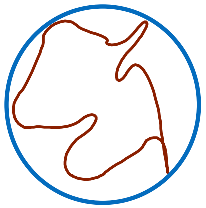
Cylinder
[Weghorst et al., 1985]



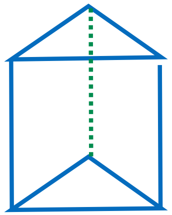
Box, AABB (R*-trees)
[Beckmann, Kriegel, et al., 1990]



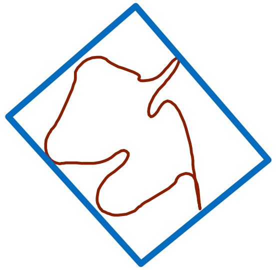
Convex hull
[Lin et. al., 2001]



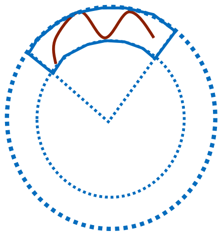
Sphere
[Hubbard, 1996]



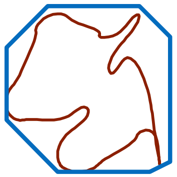
Prism
[Barequet, et al., 1996]



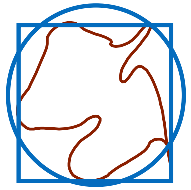
OBB (oriented bounding box)
[Gottschalk, et al., 1996]



Spherical shell
[Manocha, 1997]



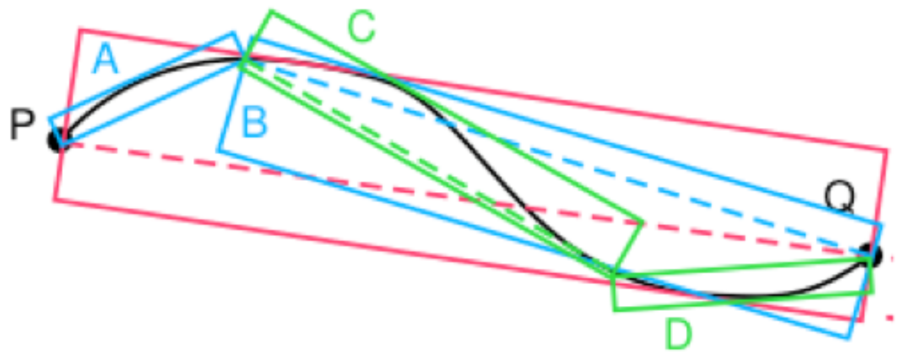
k-DOP / Slabs
[Zachmann, 1998]



Intersection of
several BVs

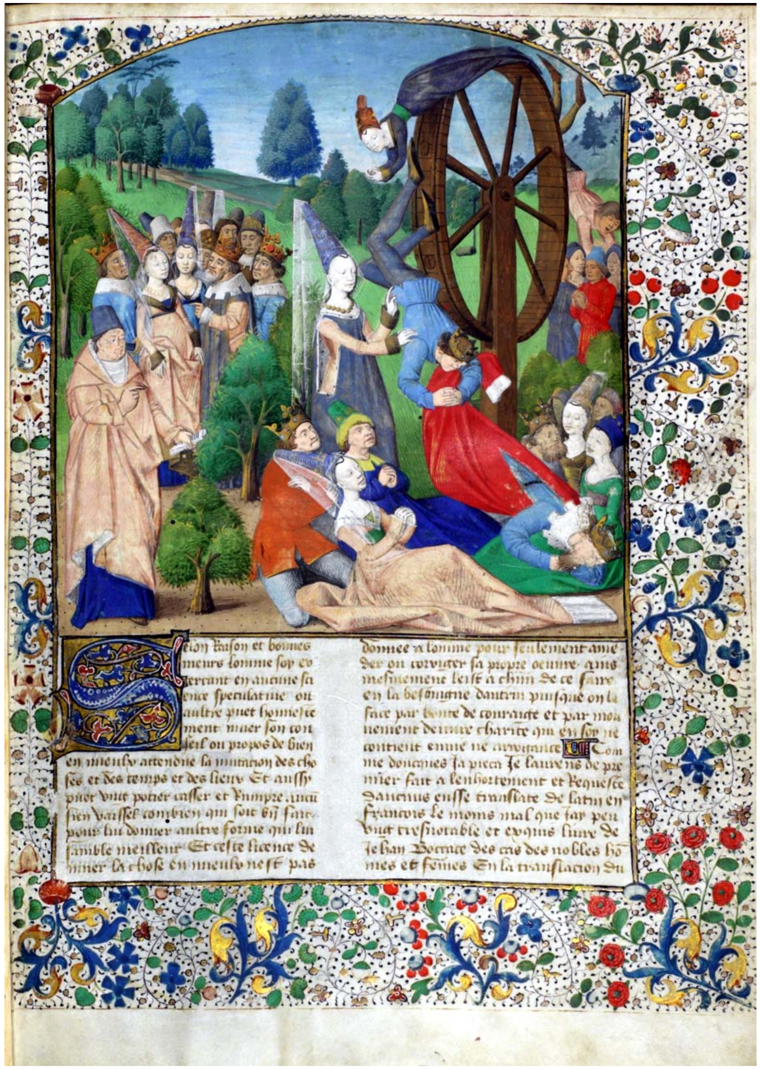
The Wheel of Re-Invention

- OBB-Trees: have been proposed already in 1981 by Dana Ballard for bounding 2D curves, except they called it "strip trees"



- AABB hierarchies: have been invented(?) in the 80-ies in the spatial data bases community, except they call them "R-tree", or "R*-tree", or "X-tree", etc.

Digression: the Wheel of Fortune (Rad der Fortuna)



Boccaccio De Casibus Virorum Illustrium Paris: 1467

Codex Buranus

Relationship Between Kind of BV and Running Time

- In case of rigid collision detection (BVH construction can be neglected):

$$T = N_V C_V + N_P C_P$$

  N_V = number of BV overlap tests
  C_V = cost of one BV overlap test
  N_P = number of intersection tests of primitives (e.g., triangles)
  C_P = cost of one intersection test of two primitives

- In case of deformable objects (BVH must be updated):

$$T = N_V C_V + N_P C_P + N_U C_U$$

  N_U / C_U = number/cost of a BV update

- As the kind of BV gets tighter, N_V (and, to some degree, N_P) decreases, but C_V and (usually) C_U increases

The Intersection Test for Oriented Bounding Boxes (OBB)

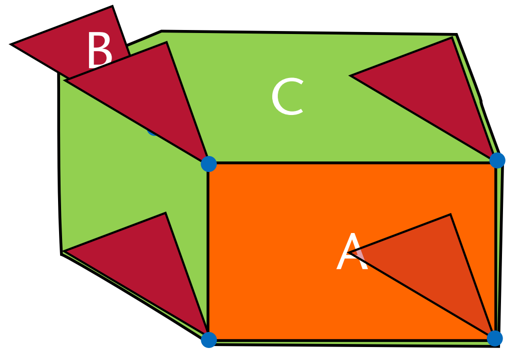
- The "separating plane" lemma
  (just a different wording of the "separating axis" lemma):
  Two convex polyhedra *A* and *B* do *not* overlap $\Leftrightarrow$
  there is an axis (line) in space so that the projections of *A* and *B* onto that axis do not overlap.
  This axis is called the separating axis.

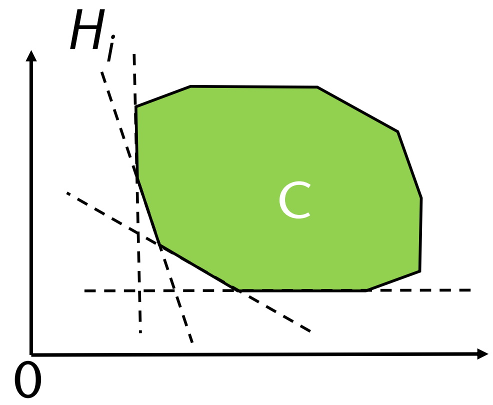
- Lemma "Separating Axis Test" (SAT):
  Let *A* and *B* be two convex 3D polyhedra.
  If there is a separating plane, then there is also a separting plane that is either parallel to one side of *A*, or parallel to one side of *B*, or parallel to one edge of *A* and one edge of *B* simultaneously.
  [Gottschalk, Lin, Manocha; 1996]

Proof of the SAT Lemma

Optional

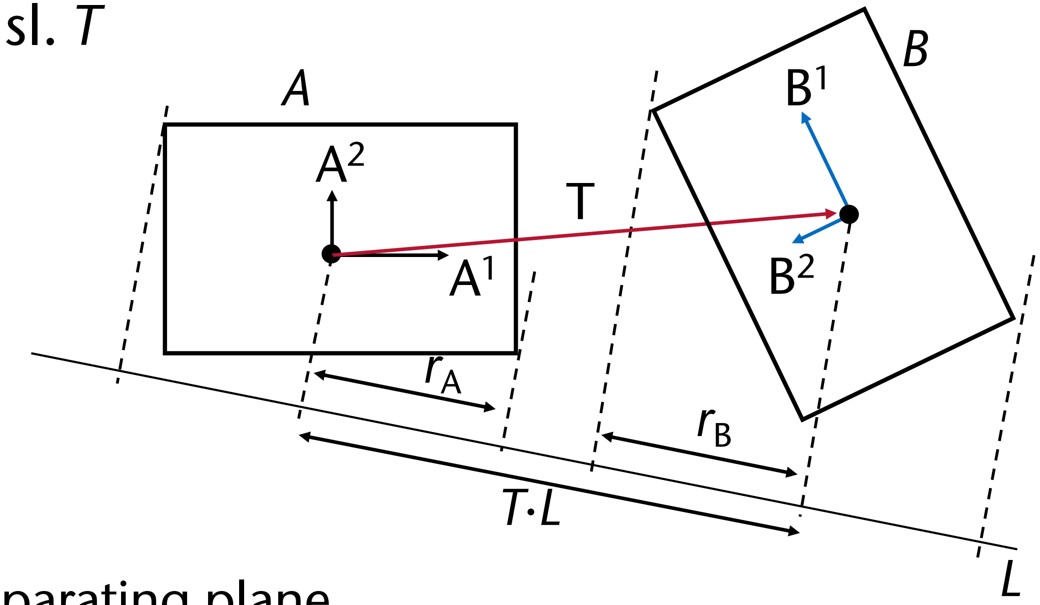
1. Assumption: A and B are disjoint
2. Consider the Minkowski sum $C = A \ominus B$
3. All faces of C are either parallel to one face of A, or to one face of B, or to one edge of A *and* one of B (the latter cannot be seen in 2D)
4. C is convex
5. Therefore: $C = \bigcap_{i=1}^{m} H_i$
6. $A \cap B = \emptyset \Leftrightarrow 0 \notin C$
7. $\exists i : 0 \notin H_i$ (i.e., 0 is outside some H_i)
8. That H_i defines the separating plane; the line perpendicular to H_i is the separating axis





Actually Computing the SAT for OBBs

- W.l.o.g.: compute everything in the coordinate frame of OBB *A*
- *A* is defined by: center *c*, axes A^1, A^2, A^3, and extents a^1, a^2, a^3, resp.
- B's position relative to *A* is defined by rot. *R* and transl. *T*
- In the coord. frame of *A*: B^i are the columns of *R*
- Let L be a line in space; then *A* and *B* overlap, if $|T \cdot L| < r_A + r_B$
  - Remark: *L* = normal to the separating plane

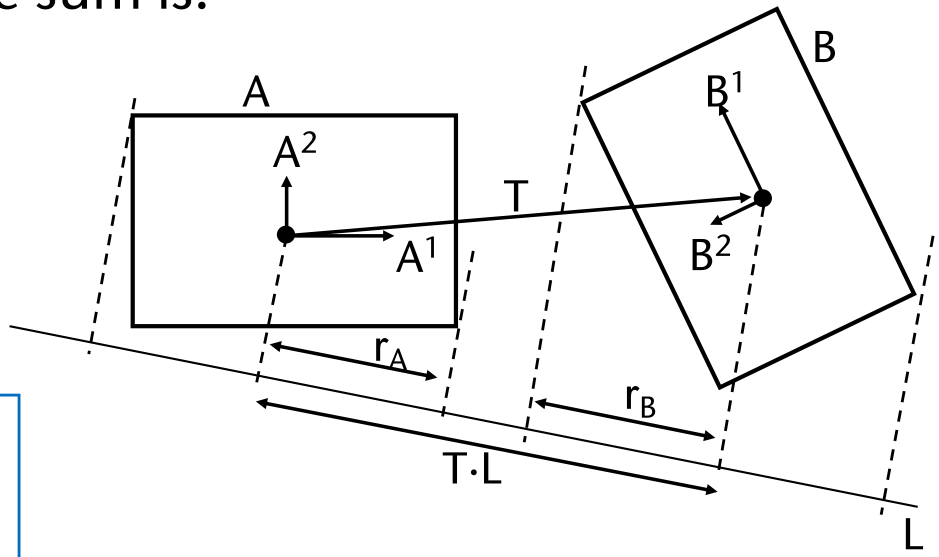


- According to the lemma, we need to check only a few special lines
- With boxes, that number of special lines = 15

- Example: $L = A^1 \times B^2$
- We need to compute: $r_A = \sum_i a_i |A^i \cdot L|$ (and similarly r_B)
- For instance, the 2nd term of the sum is:

$$\begin{aligned} & a_2 A^2 \cdot (A^1 x B^2) \\ = & a_2 B^2 \cdot (A^2 x A^1) \\ = & a_2 B^2 \cdot A^3 \\ = & a_2 R_{32} \end{aligned}$$

Since we compute everything in A's coord. frame
→ A^3 is 3rd unit vector, and
B^2 is 2ns column of R



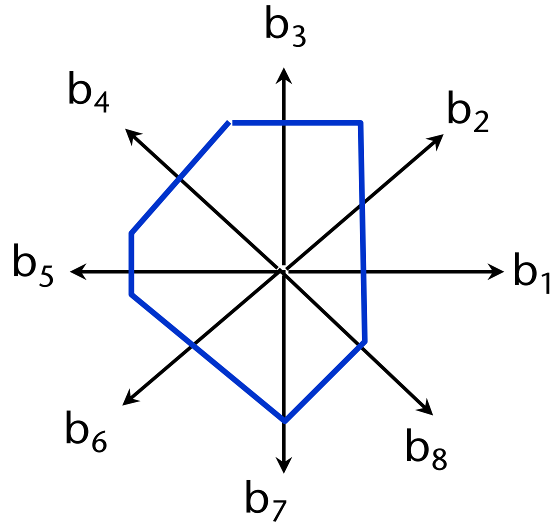
- In general, we have one test of the following form for each of the 15 axes:

$$|T \cdot L| < a_2|R_{32}| + a_3|R_{22}| + b_1|R_{13}| + b_3|R_{11}|$$

Discretely Oriented Polytopes (k-DOPs)

Optional

- Definition of k-DOPs:
  Choose k fixed vectors $\mathbf{b}_i \in \mathbb{R}^3$, with k even,
  and $\mathbf{b}_i = - \mathbf{b}_{i+k/2}$.
  We call these vectors generating vectors
  (or just generators).



A k-DOP is a volume defined by the intersection of k half-spaces:

$$D = \bigcap_{i=1..k} H_i \quad , \quad H_i : \mathbf{b}_i \cdot x - d_i \leq 0$$

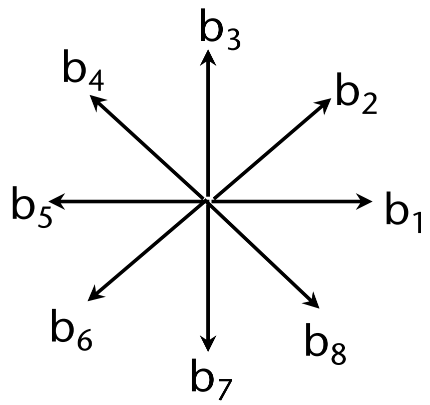
- A k-DOP is completely described by $D = (d_1 ... d_k) \in \mathbb{R}^k$

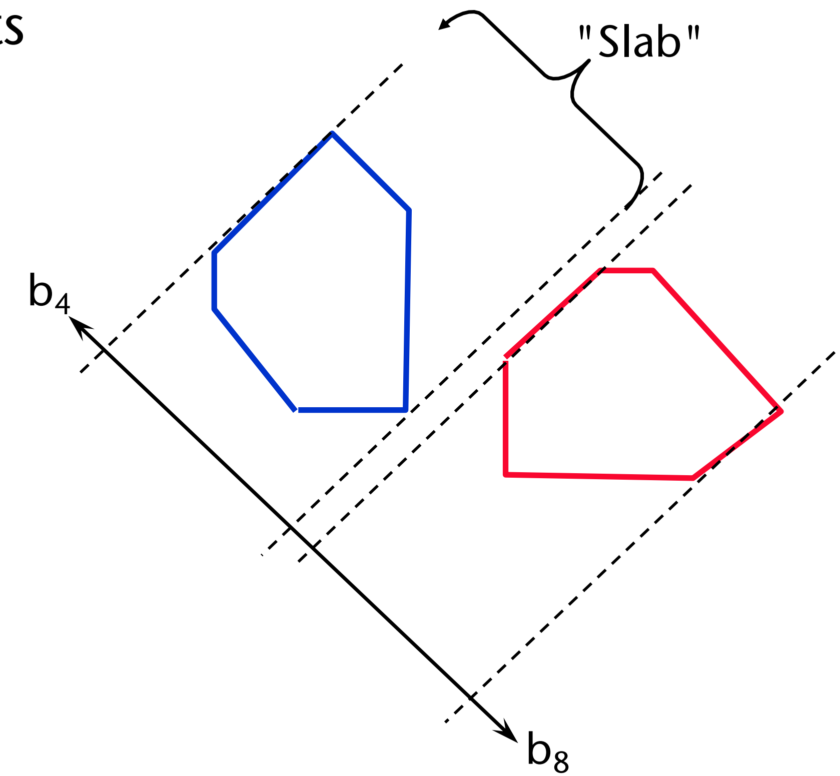
- The overlap test for two (axis-aligned) *k*-DOPs:

$$D^1 \cap D^2 = \emptyset \Leftrightarrow$$

$$\exists i = 1, \ldots, \frac{k}{2} : \left[d_i^1, d_{i+\frac{k}{2}}^1\right] \cap \left[d_i^2, d_{i+\frac{k}{2}}^2\right] = \emptyset$$

i.e., it is just $k/2$ interval tests

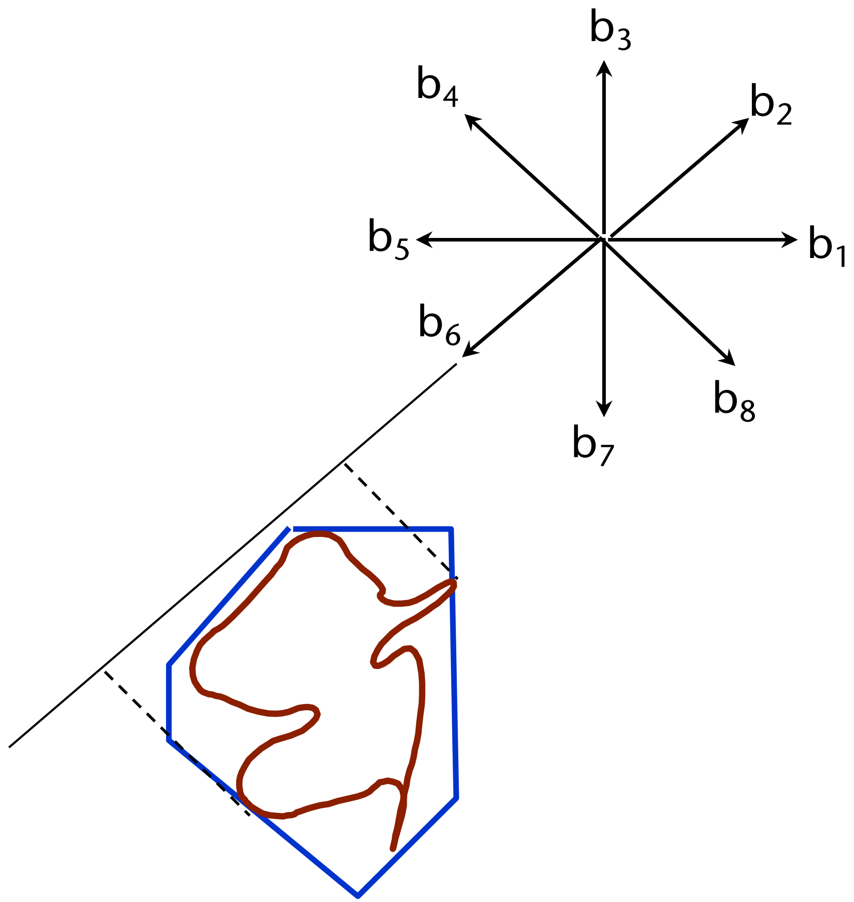




- Note: this is just a generalization of the simple AABB overlap test

- Computation of a *k*-DOP, given a polygon soup with vertices $\mathcal{V}$:
  - $\mathcal{V} = \{\mathbf{v}_0, \ldots, \mathbf{v}_n\}$
  - $D = (d_1 \ldots d_k) \in \mathbb{R}^k$
  - For each *i* = 1, .., *k*, compute

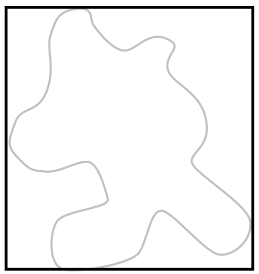
$$d_i = \max_{j=0,\ldots,n} \{\mathbf{v}_j \cdot \mathbf{b}_i\}$$



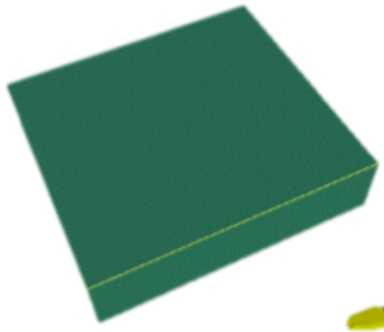
Some Properties of k-DOPs

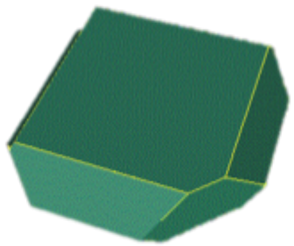
Optional

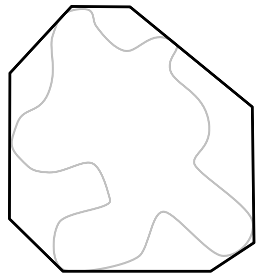
- AABBs are special DOPs
- The overlap test takes time $\in O(k)$, k = number of orientations
- With growing k, the convex hull can be approximated arbitrarily precise:



2D: $k = 4$
3D: $k = 6$

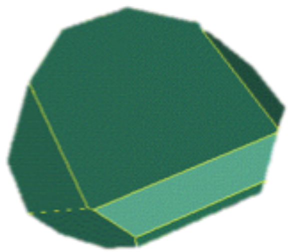


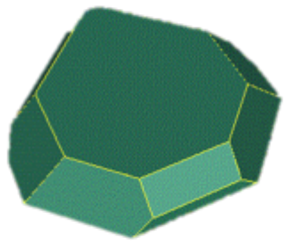




2D: $k = 8$
3D: $k = 14$

$k = 18$





$k = 26$

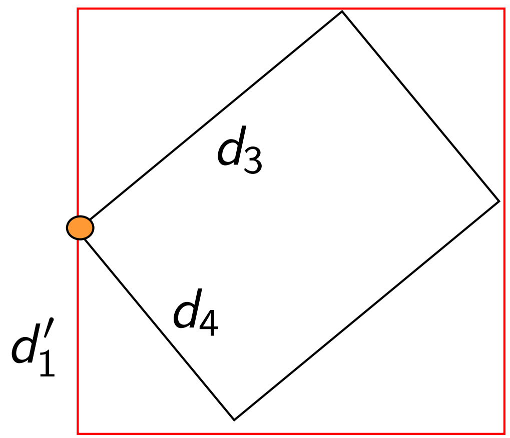
The Overlap Test for Rotated *k*-DOPs

Optional

- The idea: enclose an "oriented" DOP by a new axis-aligned one:
  - The object's orientation is given by rotation R & translation T
  - The axis-aligned DOP D' = $(d'_1, \ldots, d'_k)$ can be computed as follows (without proof):

$$d'_i = \boxed{\mathbf{b}_i \begin{pmatrix} \mathbf{c}_{j_1^i} \\ \mathbf{c}_{j_2^i} \\ \mathbf{c}_{j_3^i} \end{pmatrix}^{-1}} \begin{pmatrix} d_{j_1^i} \\ d_{j_2^i} \\ d_{j_3^i} \end{pmatrix} + \boxed{\mathbf{b}_i T},$$

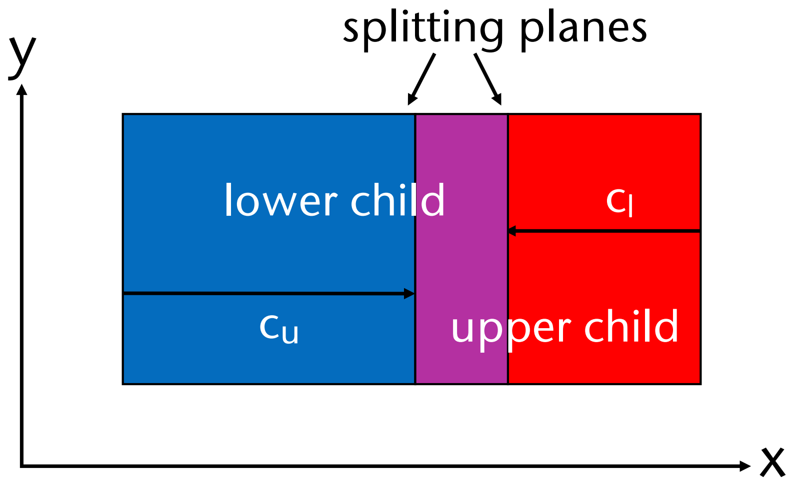
with $\mathbf{c}_j = \mathbf{b}_j R^{-1}$

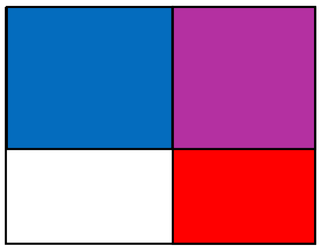


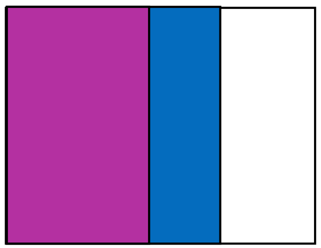
  - The correspondence j^i_l is identical for all DOPs in the same hierarchy (thus, it can be precomputed)
  - Complexity: O(k)
    - Compare this to a SAT-based overlap test

Restricted Boxtrees (a Variant of kd-Trees)

- Restricted Boxtrees are a combination of kd-trees and AABB trees:
  - For defining the children of a node B: for the left child, split off a portion of the "right" part of the box B; for the right child of B, split off a portion of the left part of B
- Memory usage: 1 float, 1 axis ID, 1 pointer (= 9 bytes)
- Other names for the same DS:
  - Bounding Interval Hierarchy (BIH)
  - Spatial kd-tree (SKD-Tree)

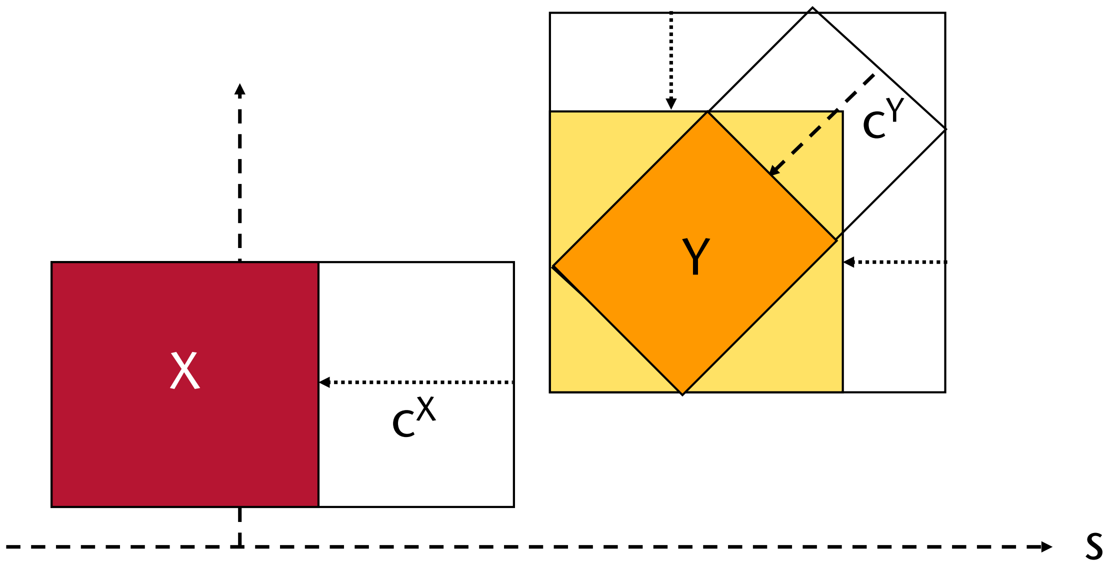






- Overlap tests by "re-alignment" (i.e., enclosing the non-axis-aligned box in an axis-aligned one, exploiting the special structure of restricted boxtrees):

  12 FLOPs (8.5 with a little trick)

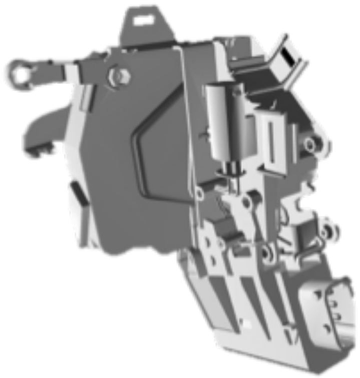


- Compare this to
  - SAT: 82 FLOPs
  - SAT lite: 24 FLOPs
  - Sphere test: 29 FLOPs

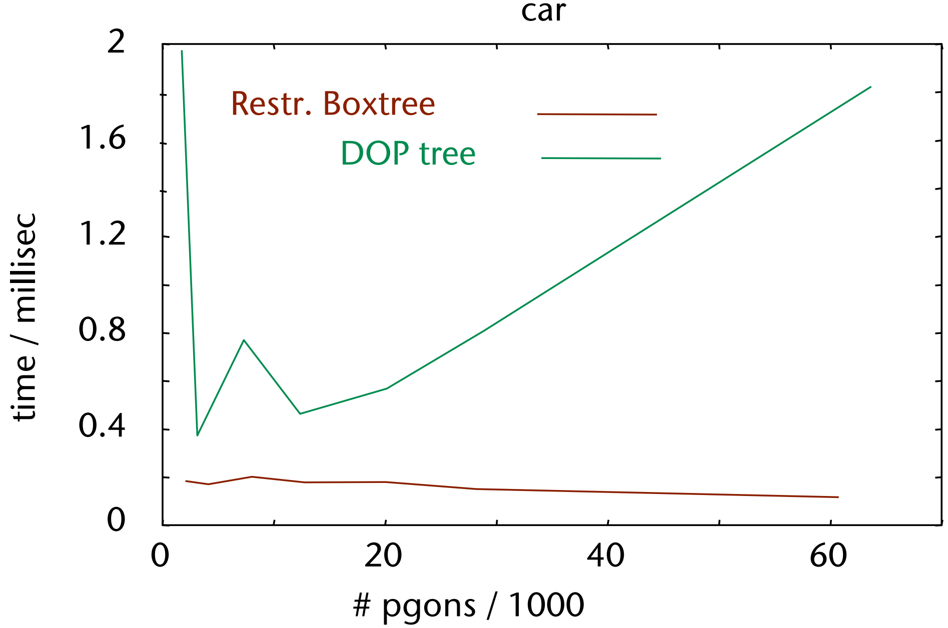
Performance

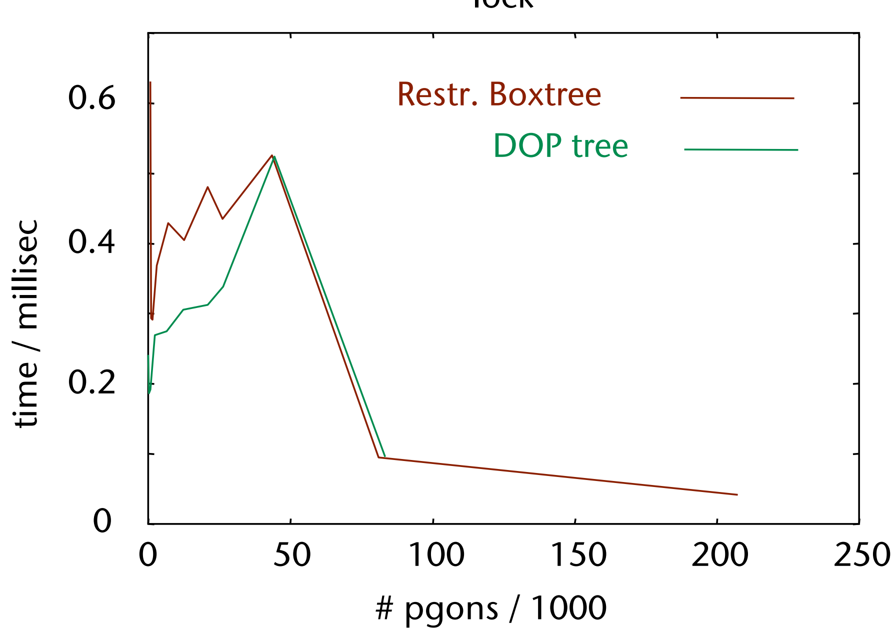
Optional

Car (courtesy VW)



Door lock (BMW)





Optional

The Construction of BV Hierarchies

- Obviously:
  if the BVH is bad → collision detection has a bad performance
- The general algorithm for BVH construction: *top-down*
  1. Compute the BV enclosing the set of polygons
  2. Partition the set of polygons
  3. Recursively compute a BVH for each subset
- The essential question: the splitting criterion?
- Guiding principle: the expected cost of collision detection incured by a particular split

$$C(X,Y) = 4 + \sum_{i,j=1,2} P(X_i, Y_j)\, C(X_i, Y_j)$$
$$\approx 4\left(1 + P(X_1, Y_1) + \cdots + P(X_2, Y_2)\right)$$

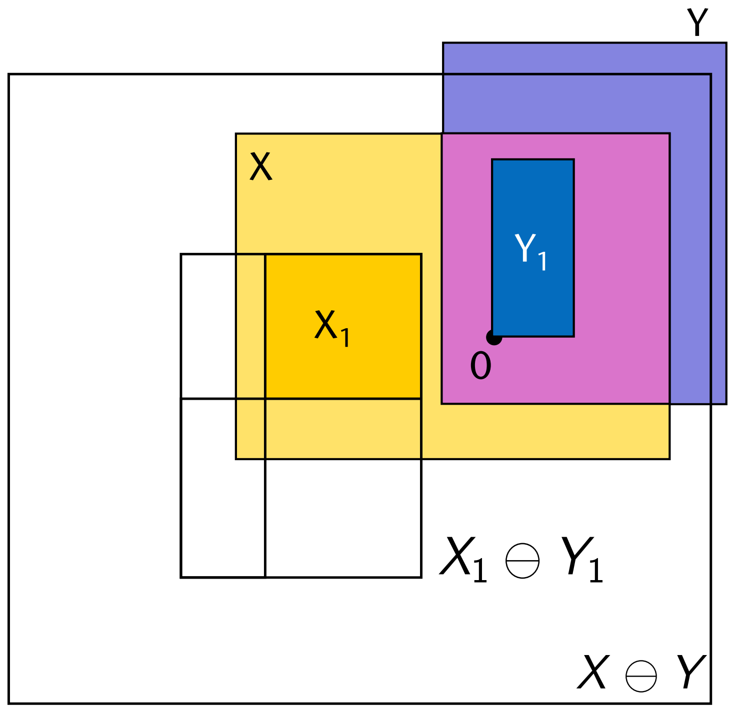
- Goal: estimation of $P(X_i,Y_j)$
- Our tool: the Minkowski sum
- Reminder:

$$X_i \cap Y_j = \varnothing \;\Leftrightarrow\; 0 \notin X_i \ominus Y_j$$

- Therefore, the probability is:

$$P(X_i, Y_j) = \frac{\#\ \text{“good” cases}}{\#\ \text{all possible cases}}$$

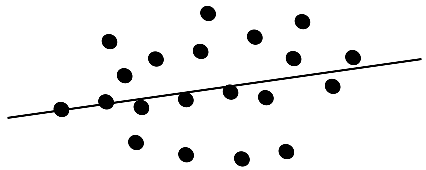
$$= \frac{\mathrm{vol}(X_i \ominus Y_j)}{\mathrm{vol}(X \ominus Y)} = \frac{\mathrm{vol}(X_i \oplus Y_j)}{\mathrm{vol}(X \oplus Y)} \approx \frac{\mathrm{vol}(X_i) + \mathrm{vol}(Y_j)}{\mathrm{vol}(X) + \mathrm{vol}(Y)}$$



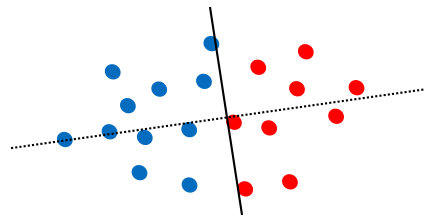
- Conclusion: for a good BVH (for coll.det.) minimize the total volume of the children of each node

Usual Algorithm for Constructing a BVH

1. Find good orientation for a "good" splitting plane using PCA



2. Find the minimum of the total volume by a sweep of the splitting plane along that axis



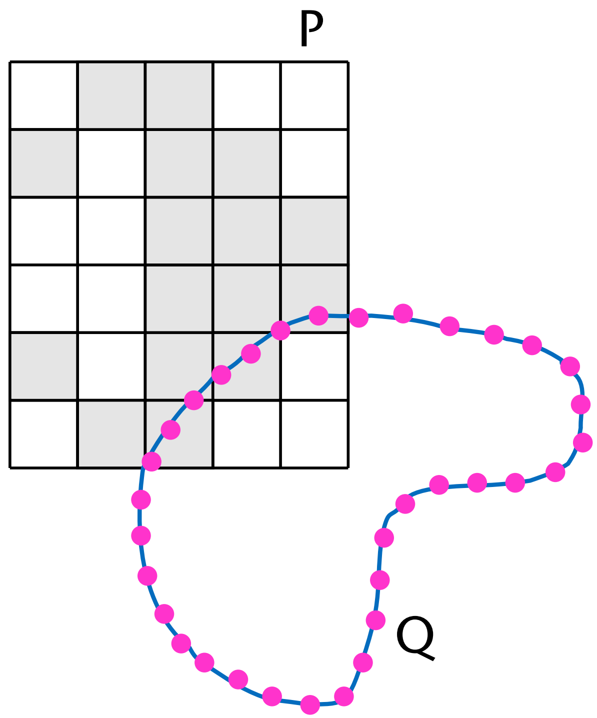
- Complexity of that *plane-sweep* algorithm:

$$T(n) = n \log n + T(\alpha n) + T((1-\alpha)n) \in O\left(n \log^2 n\right)$$

  - Assumption: splits are not too uneven, i.e., a fraction of α / (1-α) polygons goes into the left/right subtree, α not "too small"

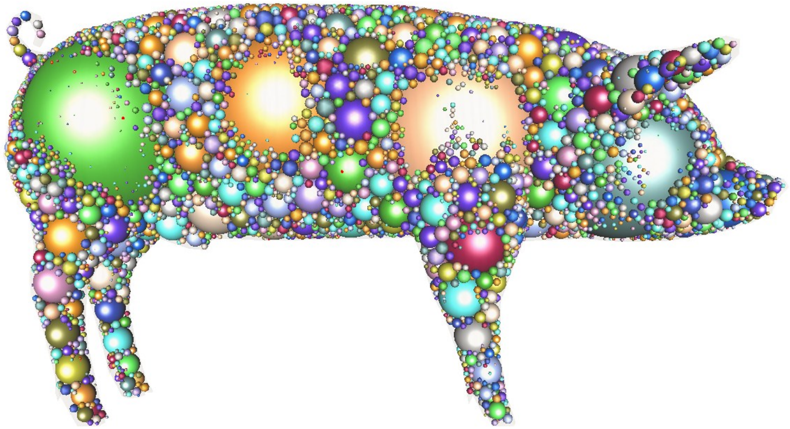
Coll.Det. Algorithm Depends on Object Representation

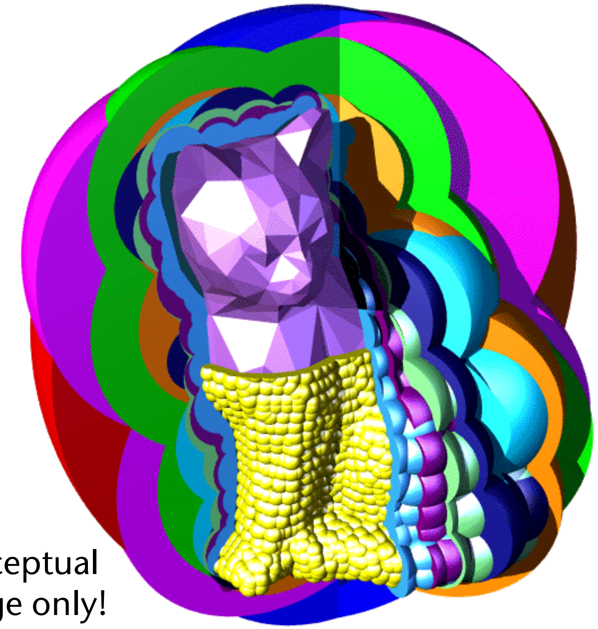
- Example: Voxmap-Pointshell
  - Objects are represented by point shell and by a voxel grid
- The fundamental operation:
  does a point hit a black voxel?
- Problems:
  - What to do in case of non-closed objects?
  - Memory consumption for all the voxels!
    - Hierarchy might help, but also slows coll.det. down
  - Collision detection is not exact (b/c of discretization)



Inner Sphere Trees: the Basic Idea

- Challenge: compute proximity, i.e., distance or measure of penetration
- Don't approximate an object from the outside; instead, approximate it
  - from the *inside* ,
  - with *non-overlapping* spheres, and
  - with as little empty volume as possible
- ➢ Sphere packing
- Build sphere hierarchy on top of *inner* spheres



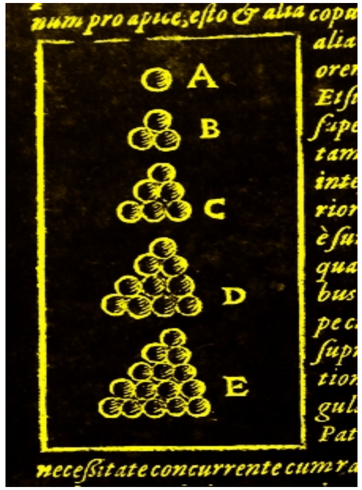


Conceptual image only!

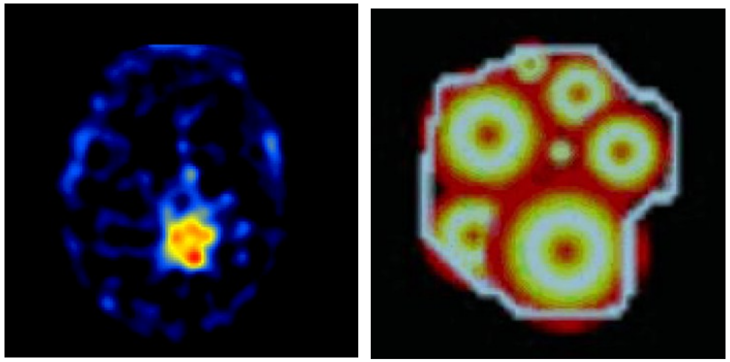
Computation of Sphere Packings

- Have a long history …

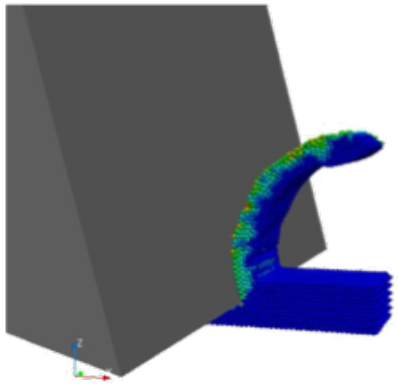
Johannes Kepler
(1571 – 1630)



- Has many applications, besides collision detection:



Radio surgery



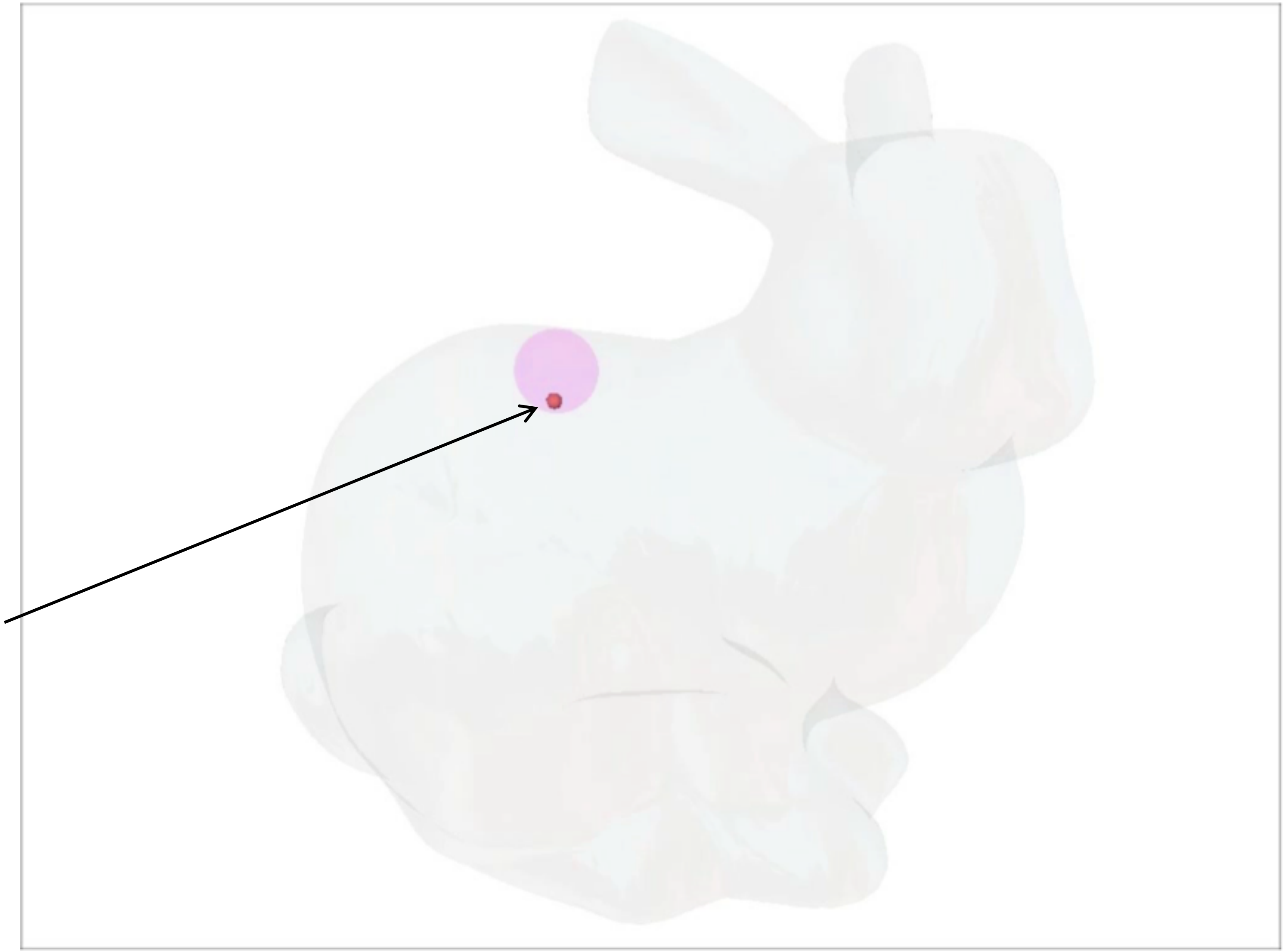
Discrete element method

Architecture

Protosphere

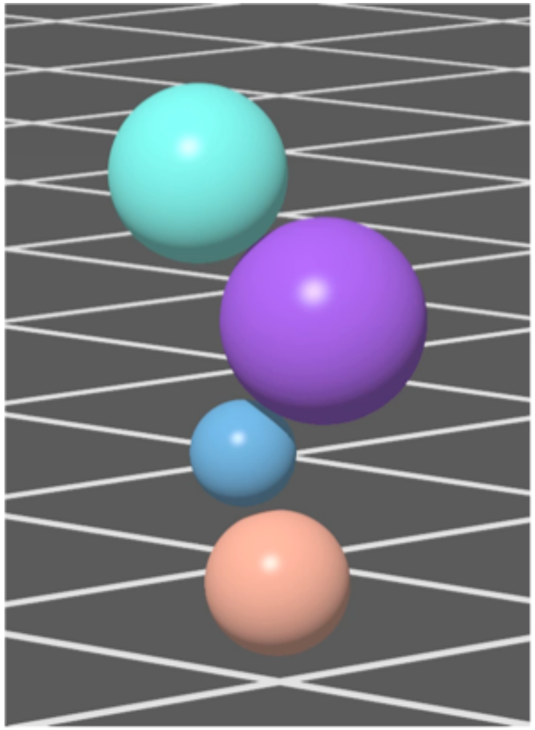
- Our requirements / variety of sphere packings:
  - Non-overlapping
  - Arbitrary radii
  - Must work for any kind of container (not just boxes)
- Optimization according to some criteria, e.g. number of spheres
- No algorithm yet for that → our approach:
  - Find inner Voronoi nodes of container object
    - (See course "Computational Geometry for CG")
    - In our case, use approximation by iterative algorithm
  - Place spheres
  - Compute new Voronoi nodes of object *plus* spheres

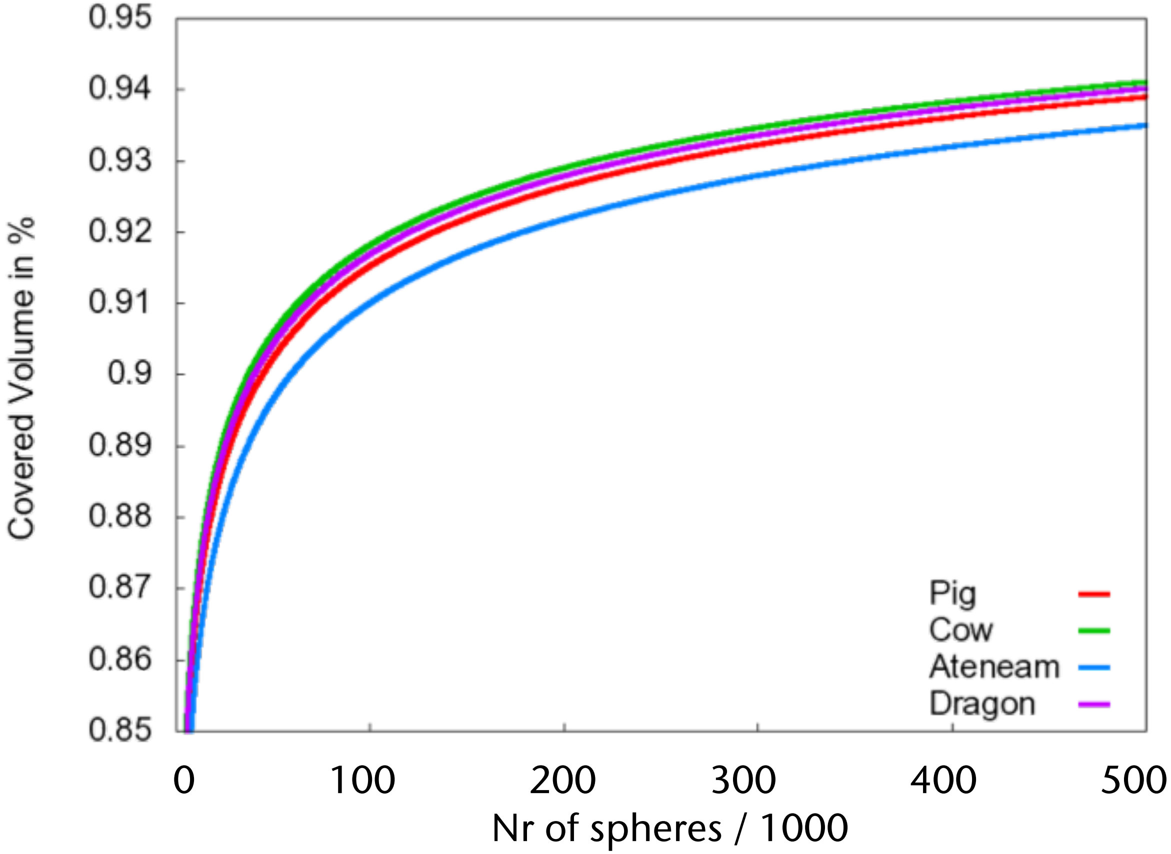
Visualization of Our Algorithm



Candidate
Voronoi node

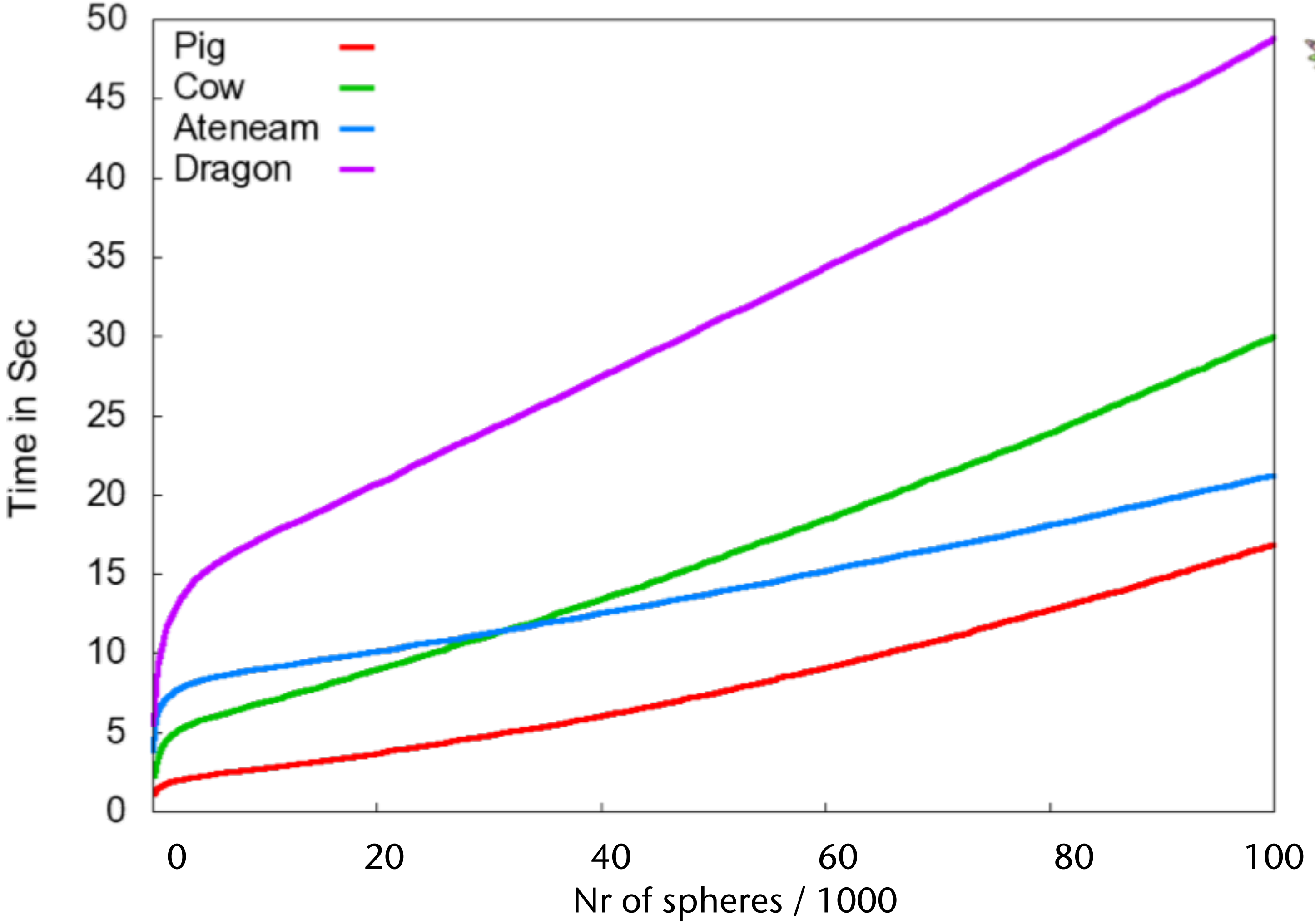
Results




0.95
0.94
0.93
0.92
0.91
0.9
0.89
0.88
0.87
0.86
0.85
Covered Volume in %
0
100
200
300
400
500
Nr of spheres / 1000
Pig
Cow
Ateneam
Dragon

Our Algorithm can be Parallelized for the GPU

Performance of Construction of Sphere Packing

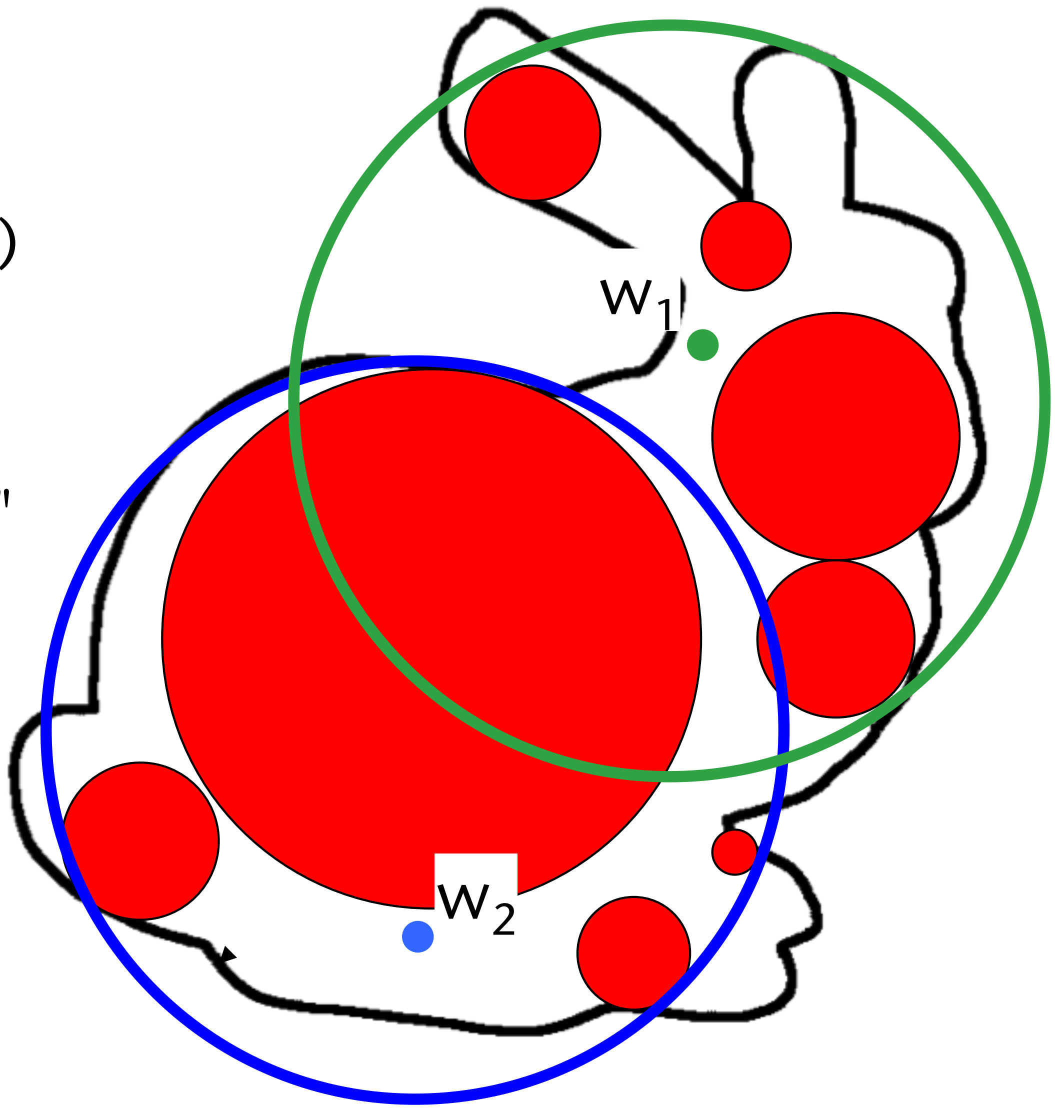


Nvidia Geforce GTX 480

Construction of Hierarchy Over Sphere Packing

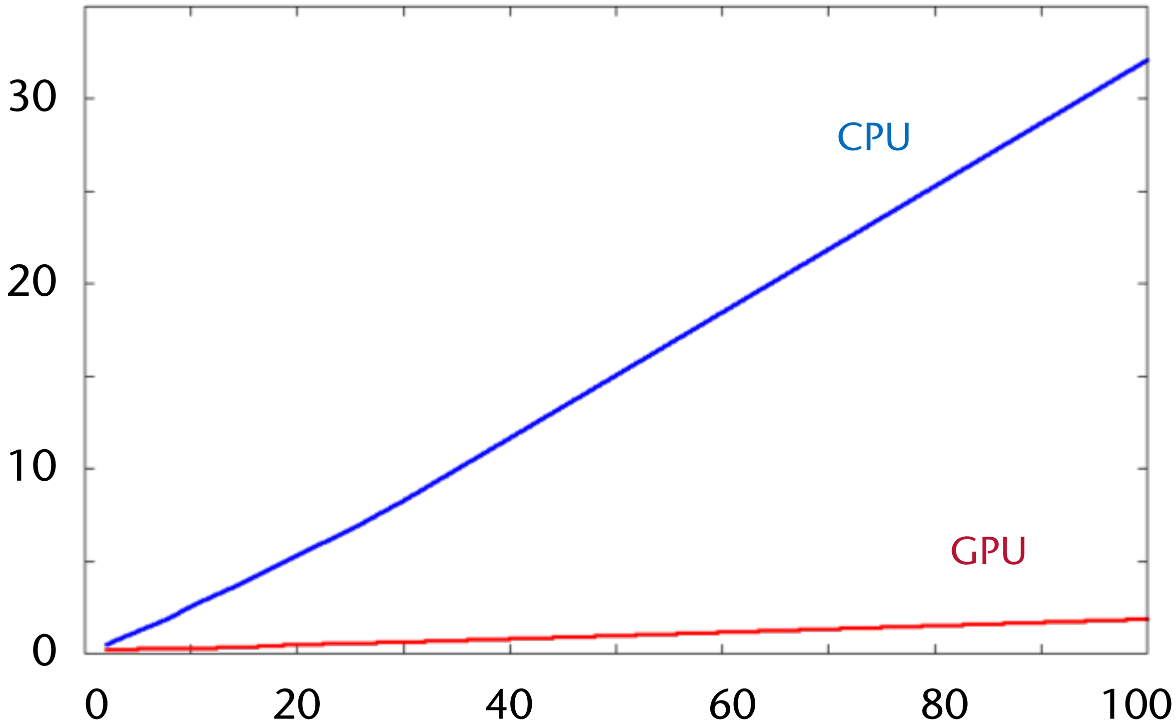
Optional

- Based on clustering method known in machine learning (batch neural gas clustering)
  - Bears some resemblance to k-means
- We can assign "importance" to spheres
- Easily parallelizable on the GPU
- Naturally generalizes to higher tree degrees (out-degree of 4-8 seems optimal)



- BNG hierarchy construction on CPU has complexity of $O(n \log n)$
- Parallelization of BNG reduces complexity to $O(\log^2 n)$

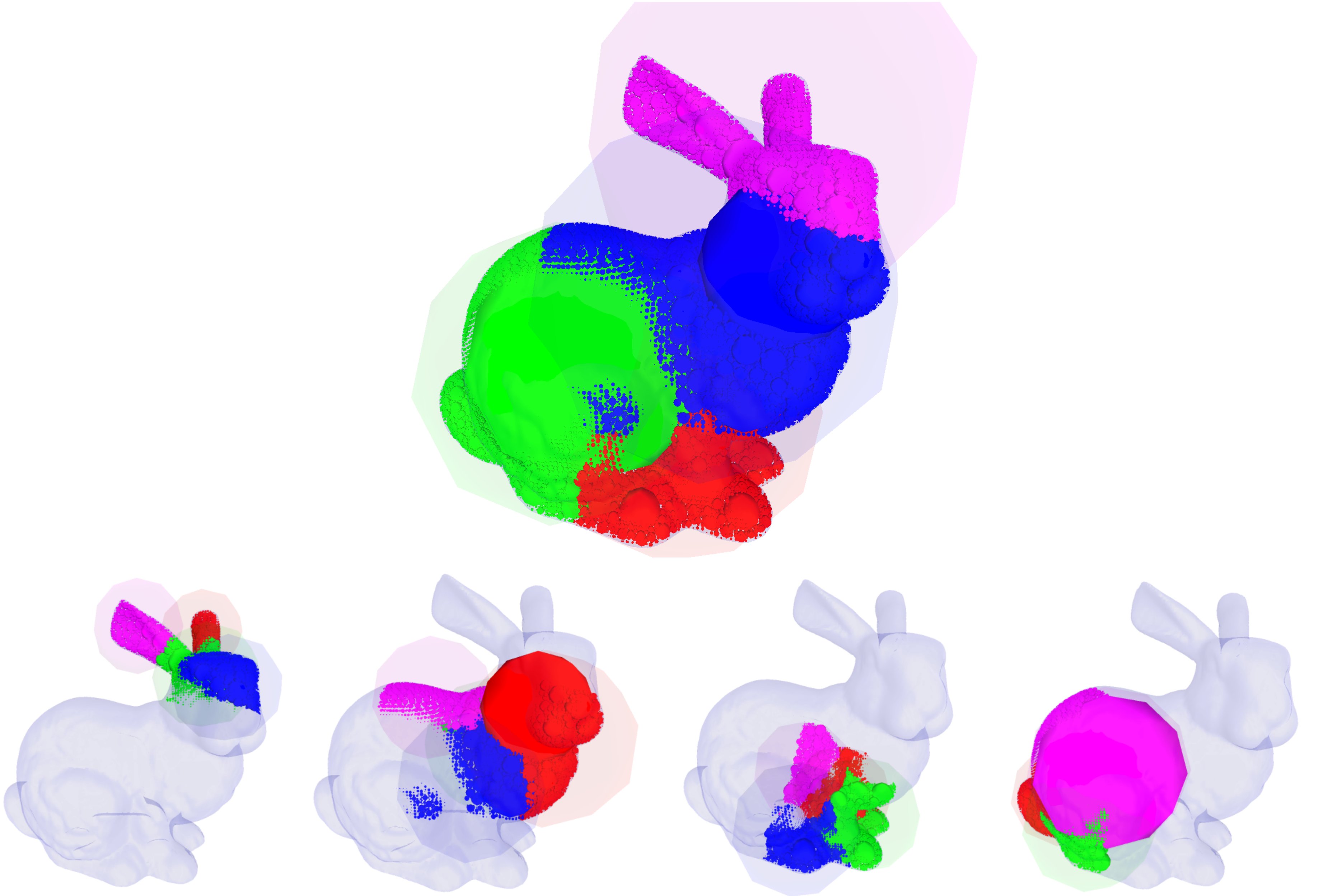
Construction time in seconds



Nr of spheres / 1000

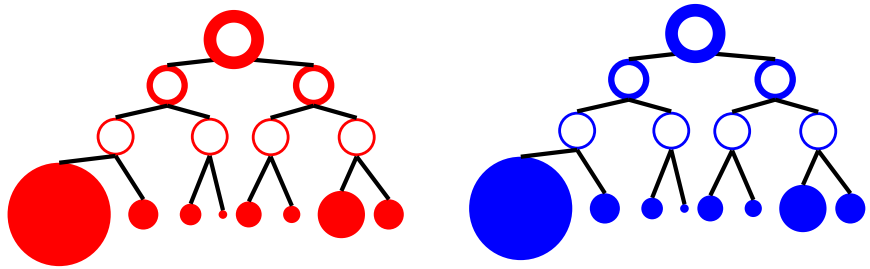
Geforce GTX 780

Examples

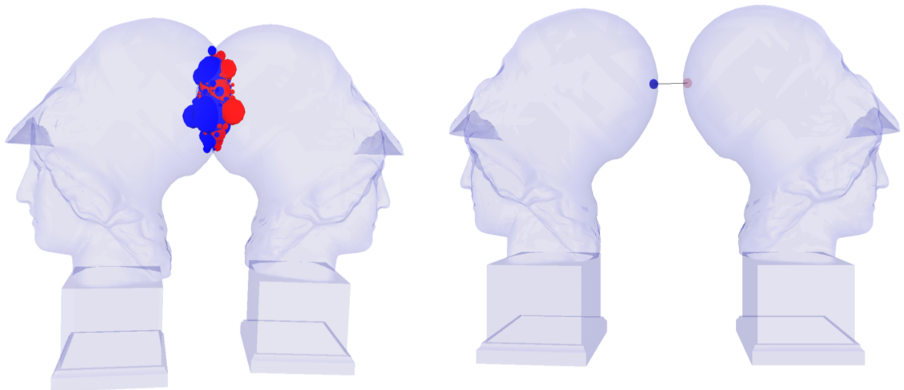


Proximity / Penetration Query Using ISTs

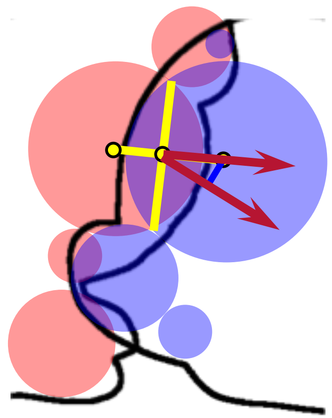
- Works by the standard simultaneous traversal of BVHs



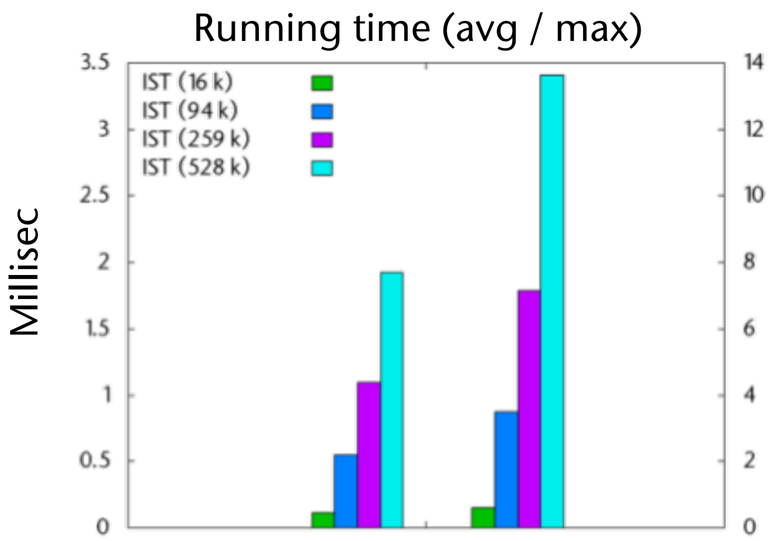
- First algo that can compute both *minimal distance* or *intersection volume* with one *unified* algorithm

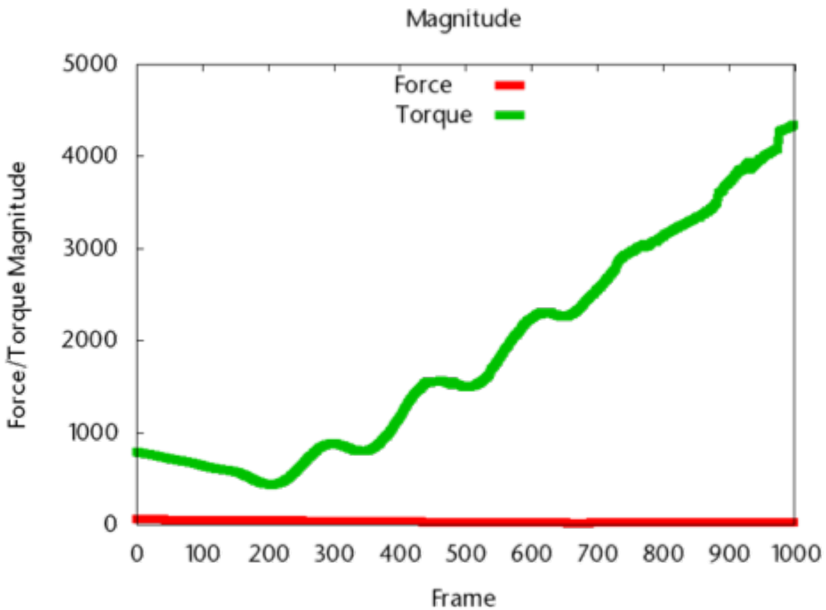


- Can compute forces and torques
  - Which can be proven to be continuous

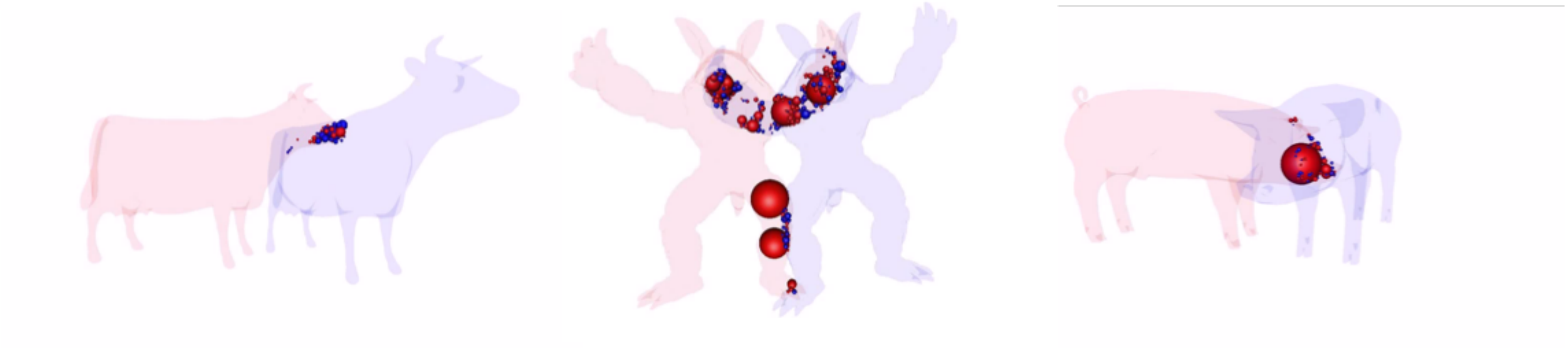


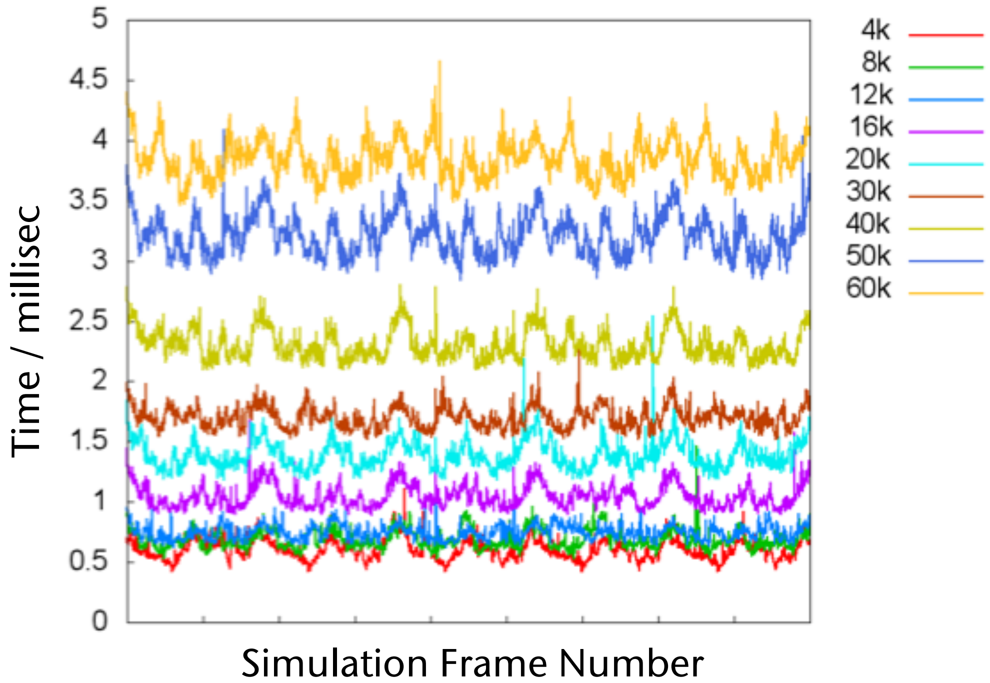
Computation Timings for the Intersection Volume

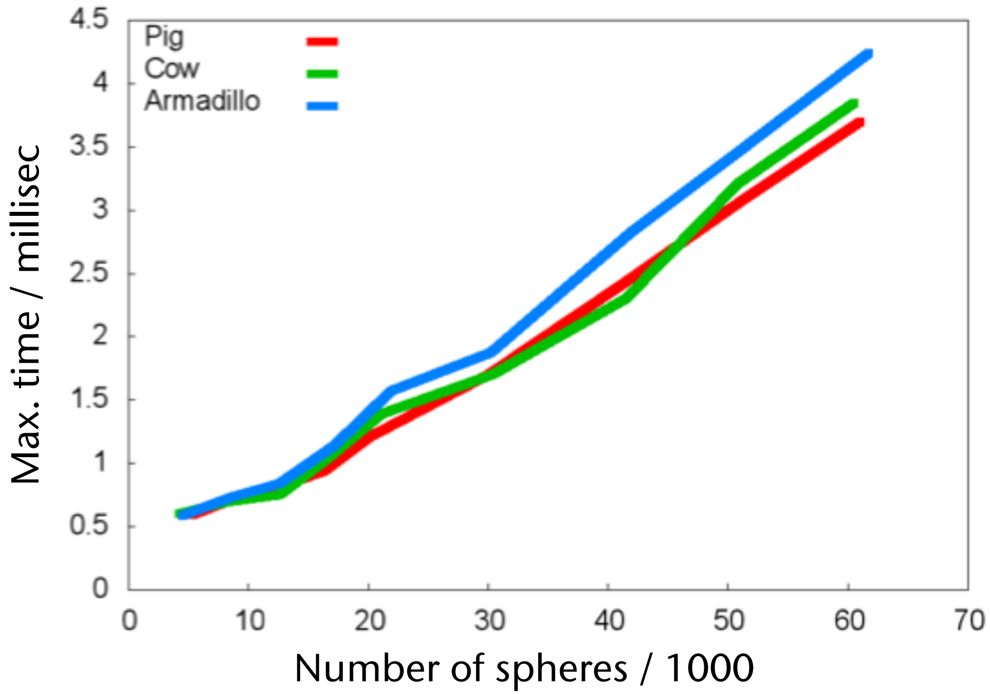




Parallel Computation Times for Intersection on GPU







Penalty Forces for Simulation/Force-Feedback

- Accumulate sphere-sphere interaction forces:
  - Linear force:

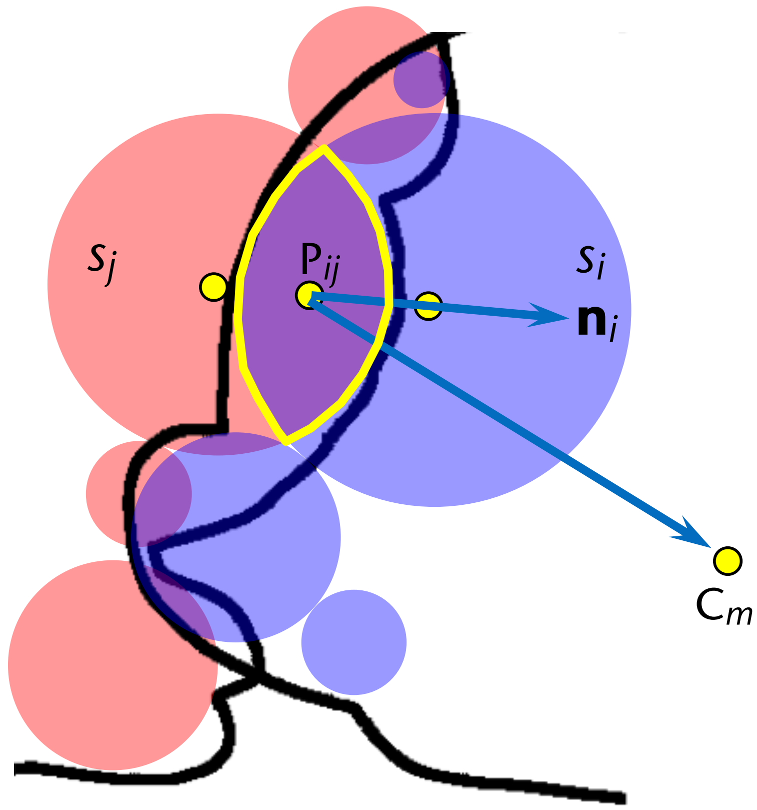
$$\mathbf{f}_{ij}^{\text{blue}} = \text{Vol}(s_j^{\text{red}} \cap s_i^{\text{blue}}) \cdot \mathbf{n}_i^{\text{blue}}$$

$$\mathbf{f}^{\text{blue}} = \sum \mathbf{f}_{ij}^{\text{blue}}$$

  - Torque:

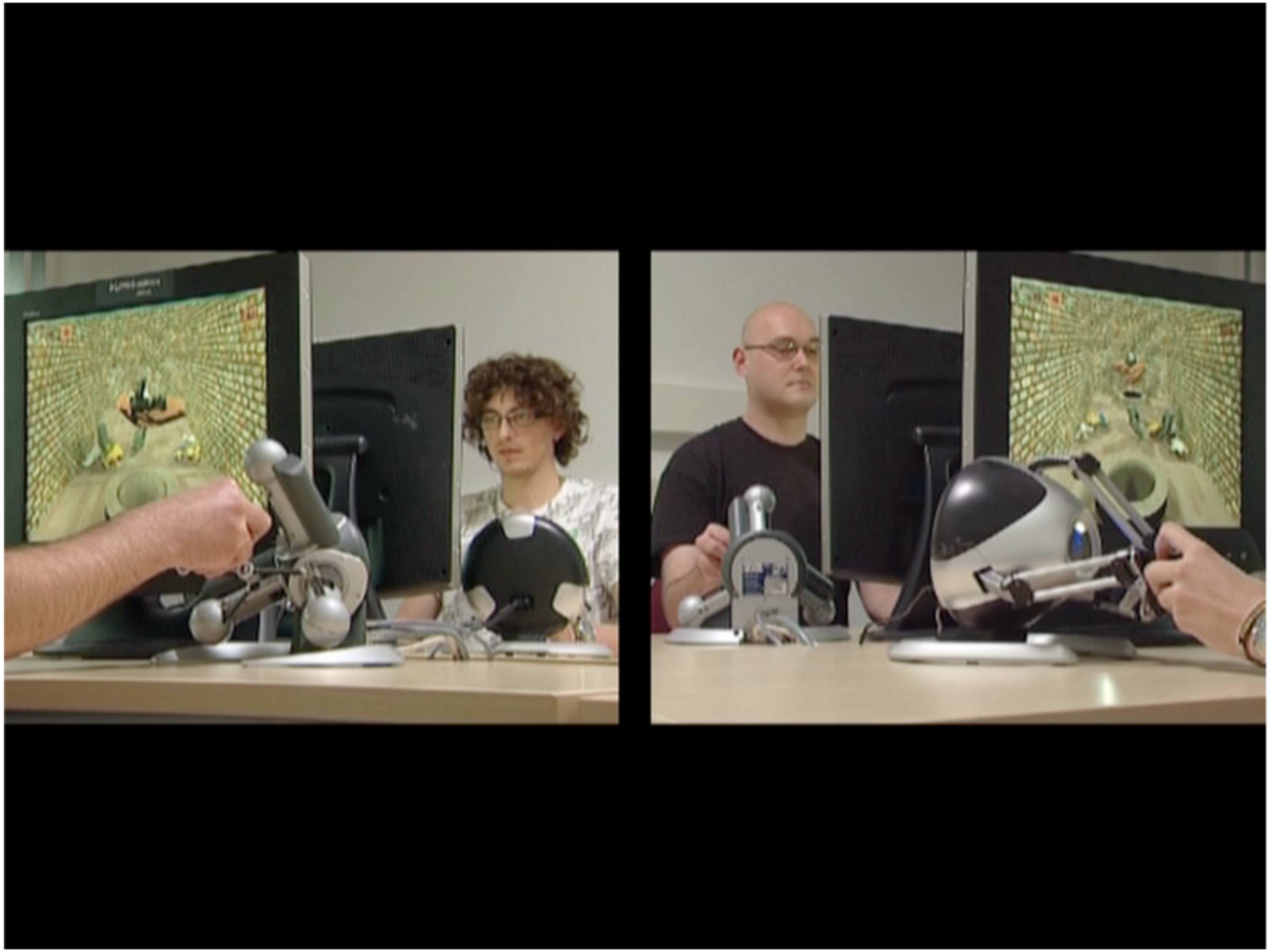
$$\tau_{ij}^{\text{blue}} = (P_{\text{ij}} - C_m) \times \mathbf{f}_{\text{ij}}$$

$$\tau^{\text{blue}} = \sum \tau_{ij}^{\text{blue}}$$



- Forces/torques an be proven to be continuous

Application: Multi-User Haptic Workspace



12 moving objects ; 3.5M triangles ; 1 kHz simulation rate ; intersection volume ≈ 1-3 msec

Master / Bachelor Thesis Topics

- Perform collision detection using machine learning
  - Use deep learning, or GLVQ (ask Barbara)
  - For riigid objects first, then deformable, or continuous collision detection